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ABSTRACT

Pupils' understanding of angle is investigated. Areas of difficulty and misconceptions are highlighted with a view to recommending improvements in the presentation of angle in primary school and remedial work to overcome problems in secondary school. The material opens with a historical and mathematical background and a brief review of research. Presented next is a survey of primary work and an analysis of the angle content of five major tests. This is followed by details of the following research of the secondary level: (1) the administration and analysis of a test on angle concepts and an examination on angle measurement; (2) a comparison of the effectiveness of two types of protractors; and (3) a presentation of nine transcripts of interviews which probed pupil understanding, along with an analysis of each presented transcript. It is felt that the findings support a number of practical recommendations for instructional improvement. Among these are findings which are thought to show clearly the advantages of full rotatable protractors over conventional half protractors in both pupil concept formation and in measurement. The document concludes with an appendix of angle worksheets. (MP)

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CHILDREN'S UNDERSTANDING OF ANGLE AT THE PRIMARY/SECONDARY TRANSFER STAGE

BY

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ABSTRACT

This research investigates pupils' understanding of angle. It highlights areas of difficulty and misconceptions in angle concepts and measurement with a view to recommending improvements in the presentation of angle in primary school and remedial work to overcome the problems in secondary school.

A historical and mathematical outline of angle definitions and measurement is given. The pedagogical implications of static and dynamic approaches to angle are described and it is seen that a fusing of these approaches is required in the development of the concept of angle.

A review of educational research points out serious misconceptions including the lack of understanding of the connection between the result of angular measurement and the attribute of angle being measured. Based on these findings a test and interview are developed. They are administered to some of the pupils in their primary school in order to ascertain their prevalent misconceptions. They reveal that most pupils have formed a purely static concept of angle and that many are unaware of the attributes of an angle which affect its size. Difficulties in the recognition of a right angle and in angle estimation are also revealed.

An analysis of primary texts indicates the reasons for many of the findings and is followed by recommendations for the improvement of the teaching of angle in primary schools.

The most surprising finding, that pupils performed far better in measuring angles with full rotatable protractors than with half protractors, is examined more closely in a test administered to all pupils once they have transferred to secondary school.

Analysis of a further test and of interviews administered in secondary school indicates the extent of each of the misconceptions previously noted. Brief recommendations for overcoming these are made and suggestions for further research into many of the problems revealed are given.

In an area where little previous research has been undertaken this work offers many insights into children's understanding of angle and the causes of their misunderstandings. It forms important conclusions for the teaching of angle which need to be tested on a wider scale and suggests many problems which require more detailed investigation.

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CHAPTER 1

INTRODUCTION

The first section gives the reasons for the choice of research into children's understanding of angle and of the pupils used as subjects.

The second section describes the intended stages of the research. It outlines the reviews of historical, mathematical and educational research literature to be undertaken prior to the administration of preliminary tests and interviews in primary schools, the analysis of primary texts and secondary school tests and interviews. It also describes the analysis of results and type of conclusions and recommendations to be made.

The third section draws attention to the difficulties encountered in primary school cooperation, taping of interviews and allocation of time.

1.1 REASONS FOR THE CHOICE OF RESEARCH

There has been a great deal more research into children's understanding of number than into their understanding of geometry. Within geometry, angle is an important concept, the nature of which differs from all other shape related concepts. It is a basic concept which children of all abilities will meet and should understand. Noticeably few research projects have been undertaken in this area yet the

large number of children possessing misconceptions about angle is immediately apparent when the topic is discussed in the classroom.

It seemed important that the sources of misconceptions should be identified and consequent remedial work recommended.

As angle is introduced in primary school and misconceptions need to be eradicated before they become too firmly believed, it was decided to concentrate on the difficulties exhibited by pupils at the primary/secondary transfer stage. Within the time available it seemed possible to perform interviews and tests with such pupils, to analyse the results, to undertake research of relevant literature and to recommend some approaches which could assist in overcoming the most serious problems.

Appropriate subjects available for the practical parts of the research would be my school's prospective first year pupils. Some of them could be interviewed and tested while in their final term at primary school and all of them could be tested during their first term at secondary school.

Recommendations could be made for the improvement of the initial presentation of angles in primary school and for remedial work in secondary school. As a unit of work on angles (see Appendix, page 1) is normally presented to first year pupils during their first term in my school, it seemed sensible to incorporate the remedial recommendations into a revised version of this unit.

1.2 OUTLINE OF THE INTENDED STAGES OF THE RESEARCH

Initially both historical and mathematical books were to be consulted for their discussion on the concept of angle. It was intended to document the evolution of both static and dynamic approaches. Definitions of angle would be detailed and compared. Also the development of angular measure was to be investigated. It was hoped that the pedagogical implications of various angle concepts and approaches could be outlined and compared, bearing in mind the applicability of these to the pupils in my London comprehensive school.

In addition the research literature on children's understanding of angle was to be consulted with a view to identifying the major misconceptions most likely to relate to those which my pupils would possess.

The analysis of these two categories of literature would result in the selection of items for a preliminary test, referred to as 'the primary test', which would reflect both important aspects of angle and serious misconceptions. The primary test and accompanying interviews (recorded on tape) were to be administered to some of my school's prospective first year pupils during their last term in primary school. It was intended that the test results and tape transcripts would highlight specific local misconceptions and their causes.

Discussions with the primary teachers would reveal the approaches to angle taught, supporting texts used and the extent of the work undertaken on angle. Following these an analysis of the angle sections in the primary texts used

would be undertaken. Together with the primary test and interview results, the text analysis would enable recommendations for improvements in the presentation of angle in primary school to be made and assist in the construction of far more detailed tests for all of the new first year pupils during their first term in my school. These tests, referred to as 'the secondary tests', were to be constructed to exhibit a wide range of the expected misconceptions and to be susceptible to statistical analysis. Accompanying interviews (recorded on tape) would also be administered to some pupils to enable the causes of errors to be probed more deeply.

The results of the secondary tests would be analysed and certain hypotheses tested statistically. Transcripts of the interviews would also be analysed.

Conclusions would be drawn regarding the most serious errors made and the misconceptions causing them. Recommendations for approaches to the concept of angle and angle measurement together with brief outlines of remedial work for inclusion in the first year angles unit would be given.

1.3 DIFFICULTIES ENCOUNTERED

It was possible to carry out all aspects of the research intended but certain difficulties were encountered in the practical parts of the research.

The head teachers of the primary schools involved were most helpful with information and in permitting the administration of tests and interviews. The primary tests were

issued to the teachers in charge of mathematics in the four main feeder primary schools but were returned completed by only one school. This led to a distortion of primary test results and consequently great care was needed in interpreting these when constructing the secondary tests. It is unlikely that this affected the final conclusions.

Problems were also experienced with the tape recorder used and, unfortunately, many of the interviews were lost. A sufficient number of interviews remained but the wider range of pupils originally interviewed would have yielded further insights into certain errors, the reasons for which were hard to ascertain.

The amount of time required to complete the project as outlined was grossly underestimated and consequently it was not possible to administer interviews to follow up all serious errors noted in the secondary tests. The recommendations for remedial work were given in a much briefer format than originally intended but still in sufficient detail to aid in the construction of a more efficient first year angles unit.

CHAPTER 2

THE CONCEPT OF ANGLE

The emergence of the concept of angle has been piecemeal. It has evolved through both geometry and measurement. Even now there are different approaches to the concept.

The wide variety of approaches is illustrated both historically and mathematically. Definitions of angle, which are related to approaches to geometry, are discussed in the first section. Angular measure, which evolved separately and from practical requirements, is discussed in the second section. The third section analyses the relative merits of different angle concepts and approaches from a pedagogical viewpoint and makes recommendations for children's early experiences of angle.

2.1 ANGLE DEFINITIONS

There has never been one universally accepted definition of angle. The evolution from various forms of angle to the present day notion is outlined. Modern definitions are categorised as 'static' or 'dynamic' and examples of each are given.

The numerous ancient definitions of angle indicate the wide variety of approaches to this concept even at its inception. When or where it first emerged is not clear. Although angles arose naturally in astronomy and geometry

attention was initially focussed on the times or lengths involved rather than on the angles themselves. One view of how the concept emerged, expressed by Morris Kline [1, p.31], is that it came from observation of the angles formed at the elbows and knees. In support of this view he cites the use of the word 'arm' of an angle in English and 'leg' (das Bein) of an angle in German.

The early Greeks possessed the notion of angle and sought to define it. They considered angles in solids and on both plane and curved surfaces. An elaborate classification due to Geminus [Heath, 2, p.178] reveals that plane angles formed by two straight lines were just one of the many types of angle considered. Other types of angles on surfaces were between curves or between lines and curves. Most Greek definitions attempted to embrace all types of angle.

> Apollonius [Heath, 2, p.176] defined an angle as, 'a contracting of a surface or a solid at one point under a broken line or surface'. An early idea of a plane angle, as described by Aristotle, was that of a deflection or breaking of a single line.

Many early definitions were abstruse. Heath [2,p.177] interprets Plutarch's definition as an attempt to obtain the rate of divergence of two lines at their point of intersection and Carpus' definition as a precursor to the notion of angle as a rotation. Both of these definitions related to all types of angle and considered angle as a form of 'distance', no alternative vocabulary being available at the time to describe this notion more clearly.

The earliest widely accepted definitions related to angles in a plane and were made by Euclid. He defined an angle in terms of the relationship between two lines rather than in terms of a distance or the configuration of one line. His definitions 8 and 9 from The Elements as translated by Heath [2, p.176] are given below.

> 8. A plane angle is the inclination to one another of two lines in a plane which meet one another and do not lie in a straight line.
>
> 9. And when the lines containing the angle are straight, the angle is called rectiiineal.

The general definition of a plane angle (definition 8) was formulated to include angles between curves and between lines and curves in addition to angles between two lines. It is circular as it presupposes a basic notion of angle in that no previous method exists for finding the inclination between two lines or, in other words, the difference between two directions, as direction is not a magnitude. It excludes zero and straight angles and the assumed formation of only one angle by any one inclination also excludes reflex angles.

The rectilineal angle (definition 9), which was a special case for the early Greeks, is now the generally accepted form of a plane angle. It was the one used by Euclid himself throughout The Elements.

Greek philosophers discussed the nature of angle and debated whether an angle should be classified as a quantity, quality or relation. Quantities possess magnitude and can thus be compared for equality. Plutarch and Carpus defined

angle as a quantity. Qualities describe the form of objects which can then be compared as possessing more or less of the quality. Straightness and curvature are qualities. Aristotle considered deflection to be a quality. Apollonius' definition (p. 7) treats angle as a quality. A relation describes the connection between items such as the two rays which form an angle. Euclid's definition (p. 8) falls into this category but from his work it is clear that he regarded angles as magnitudes. Proclus [Heath, 2, p.178] stated that an angle possessed aspects of each category.

Most of the definitions of a plane angle from ancient times onwards were classified by Schotten in his Inhalt und Methode des planimetrischen Unterrichts, II, 1893, pp 94-183 into three groups. Heath [2, p.179] translates these as follows.

> '1. The angle is the difference of direction between two straight lines.
> 2. The angle is the quantity or amount (or the measure) of the rotation necessary to bring one of its sides from its own position to that of the other side without its moving out of the plane containing both.
> 3. The angle is the portion of a plane included between two straight lines in the plane which meet in a point.'

Definitions in groups 2 and 3 will be referred to as 'dynamic' and 'static' respectively.

Euclid's definition (p. 8) falls into group 1. Modern definitions, such as those of Choquet and Hilbert fall into group 2 or 3. Most modern static definitions refer either to the union of the sets of points forming two rays or to the intersection of two half planes. The static

approach is utilised in Euclidean geometry in which a central concept is congruence and consequently the triangle is an important figure. Euclid compared angles, lengths and hence triangles by superposition [Heath, 2, p.224]. The dynamic approach is utilised in transformation geometry in which isometries are a central concept and allow comparison of angles.

David Hilbert [3, p.11] uses a static definition of angle in terms of rays, which he has previously introduced as the totality of all points of a line which lie on one side of a certain point on the line. Points, lines and planes have been introduced earlier as 'sets of objects' [3, p.3]. His angle definition is as follows.

> 'Let α be a plane and h, k any two distinct rays emanating from O in α and lying on distinct lines. The pair of rays h, k is called an angle and is denoted by $\sphericalangle$ (h,k) or by $\sphericalangle$ (k,h).'

It excludes zero, straight and reflex angles.

Gustave Choquet [4, p.79] uses a dynamic definition of angle in terms of rotations of the plane Π. His approach is based on the central concept of isometries. The set of linear isometries of the plane which preserve orientation (sense) are called rotations and form a commutative group. His definition is as follows.

> 'For every O $\in \Pi$, a rotation about O is called an angle with vertex at O. If (A_1,A_2) is a pair of half-lines whose origin is O, the rotation about O taking A_1 to A_2 is called the angle formed by the pair; it is written A_1A_2.'

Zero, straight and reflex angles are included, but angles are only distinguishable mod 2π. Angles at different vertices are compared by translation [4, p.79].

2.2 ANGULAR MEASURE

Early examples of the use of the right angle as a unit of measure are given. The need for and gradual development of a smaller unit, the degree, is outlined as are the common opinions regarding the origin of the number of degrees in a full turn and of the subdivision of degrees into minutes and seconds. It is seen that angles as such were overlooked in early astronomical and geometrical calculations. The relative priority of the notions of arc length and angle are discussed, the creation of the radian is noted and the mathematical problems of angle measurement are commented upon.

The first commonly used unit of angular measure was the right angle. It was mainly applied to static angles in practical situations such as building and marking out plots of land.

Lancelot Hogben [5, p.45] stresses the importance of plumb line devices and the concept of perpendicularity even before the time of the Greeks and the consequent emergence of the right angle as the first fundamental unit of angular measurement.

The right angle became a fixed standard against which other angles were compared and from which acute and obtuse angles were defined. In early astronomical records, where rotations were being considered, the right angle unit was also used although it gave rise to many fractional results.

The inbuilt natural measure of rotation facilitated easy and universal description of numbers of full turns and simple fractions of turns. In astronomy where comparatively long times were taken for complete revolutions records were desired of short term observations and hence of small fractions of a rotation. Consequently a smaller unit, which would be more appropriate in this context, was required.

Between 4000 and 3000 B.C. the Babylonians had become interested in astronomy for its own sake and for its relation to religious concepts, the calendar and the seasons which governed their pattern of crop cultivation. They amassed tables of data from their observations and used these for calculations and predictions. About 2000 B.C. they developed a number system using the concept of place value with a base of 60 in which both whole numbers and fractions were representable.

In spite of these parallel developments, no records of Babylonian observations or calculations using subdivisions of a circle into degrees exist and it is not clear from where the idea of creating 360 initial divisions of a circle originated. P.S. Jones [6] suggests that 360 may have arisen as a rough estimate for the number of days in a year as twelve thirty day months. Hogben [5] supports this view as early Chinese geometers had divided the circle into 365 parts,

assuming this to be the exact number of days in a year.

Although the Egyptian and Babylonian civilisations made astronomic calculations and tables relating to lengths of triangles (such as $\sec^2 x$ and $\tan^2 x$ on Plimpton 322) they did not introduce 'a measure of angles in the modern sense' [Boyer, 7, p.38]. Their results stemmed from notions of ratio of lengths in geometric figures rather than from notions of angle.

The Greeks were the first to undertake systematic studies of astronomic data and of relationships between angles or their corresponding arcs in circles and the lengths of chords subtending them. Initially their results were expressed in right angles or quadrants and fractions of these units. Hipparchus (circa 150 B.C.) was the first to draw up tables for corrsponding chord and arc lengths using a series of angles based on the subdivision of the circle into 360 parts. He had probably been influenced by Hypsicles (circa 180 B.C.) who was the first Greek astronomer to divide the circle of the zodiac into 360 parts and who had followed the Chaldean practice of dividing it into twelve signs each of which had thirty parts. Boyer [7, p.180] states that it was Hipparchus' adaptation of this specific usage to the circle in general that initiated the popular and systematic use of the degree.

Ptolemy created detailed astronomic and trigonometric tables. In his Almagest (circa A.D. 150) he tabulated, correct to the nearest second, chords of arcs from $\frac{1}{2}^\circ$ to 180° for every $\frac{1}{2}^\circ$. Astronomers used the tables for more than 1000 years. To draw up his tables, which were based on

lengths of lines rather than the present day notion of trigonometric ratios, he required conventions for the subdivision of both the diameter and the circumference of the circle. He chose to keep the 360 degree division of the circumference which had been in use since the time of Hipparchus and selected subdivisions of the degree which were almost certainly prompted by the convenience of the Babylonian sexagesimal fractions which could be tabulated and manipulated far more efficiently than the Egyptian unit fractions or Greek common fractions. According to the Latin translation he subdivided his degrees into sixty 'partes minutae primae' and each of these into sixty 'partes minutae secundae' from which the words 'minutes' and 'seconds' were derived. [Boyer, 7].

The degree seems to have evolved as a measure of two almost synonymous quantities, the length of an arc on a certain circle and the angle at the centre subtending it. Although used in tables of static quantities, arcs and chords, the unit of angular measure stemmed from the consideration of a dynamic notion, that of the rotation of the planets on assumedly circular paths.

Commentaries on 'dividing the circle into parts' do not clarify whether it is the circumference or the angle at the centre which is deemed to have been divided. Evidence of Greek geometrical constructions, such as the subdivision of the circumference of the circle into six equal parts using its radius as a chord, seem to imply that any subdivision of a circle into equal sectors would be performed by first subdividing the circumference and then connecting

these points to the centre of the circle. It would then be observed that the angles at the centre were equal. Notions of arc length seem to be prior to those of any static or dynamic concept of angle at the centre of the circle.

The degree has now come to be thought of as a unit of angular measure rather than an arc of a circle. It is only in comparatively recent times that an angular unit based on arc length has come into use. The radian was defined by Thomas Muir and James T. Thomson [P.S. Jones, 6] in 1873 as the angle at the centre of a unit circle subtending an arc of unit length. This unit, consciously and carefully created by a mathematician and a physicist, yielded great simplifications in both fields, particularly in the differentiating and integrating of trigonometric functions. The idea had actually been in use in both pure and applied mathematics for nearly two hundred years prior to the formal naming of the radian [Jones, 6].

Mathematically the measurement of angles poses problems as it differs greatly from the measurement of such quantities as length, area and volume. The units of length are arbitrary and relative whereas angle has an inbuilt natural measure of a full turn and consequently the sum of two non-zero angles can equal zero.

When the linear dimensions of a geometrical figure are multiplied by a constant the geometrical properties of the figure are not altered. When the angles of a triangle are, say, doubled the triangle is not merely distorted but destroyed. Torretti [8, p.317] comments on the debates about this difference between linear dimensions and angles

and quotes the following remarks relating to it made by Bertrand Russell.

> 'If we admit the logically relative nature of every magnitude, I cannot see why the argument should apply only to linear dimensions and not to angles which are magnitudes as well'.

To define an angular measure a mapping needs to be constructed from the set of reals, R, onto the set of angles A (see Choquet [4], p.98). In A there is an angle ω such that $2\omega = 0$. Clearly $0 \in R$ maps onto $0 \in A$. There exists an $x \in R$, where $x \neq 0$, which maps onto $\omega \in A$, therefore $2x$ maps onto 2ω i.e. 0. Hence the set of real numbers which maps onto zero (which cannot be R or only {0}) must be a set of multiples $\{n\ a\}$, $(n \in Z)$. If degrees are chosen $a = 360$ and $\omega = 180$; in radians $a = 2\pi$ and $\omega = \pi$. This mapping enables angles greater than 2π to be distinguished.

2.3 PEDAGOGICAL IMPLICATIONS

The unsuitability for school pupils of axiomatic approaches to angle is outlined. The need to integrate aspects of both static and dynamic approaches with measurement in developing a concept of angle is discussed. Three main angle concepts put forward by Freudenthal, together with their pedagogical implications, are described. Recommendations are made for children's early experiences of angle.

Rigorous axiomatic foundations for angles, for example those of Choquet and Hilbert, now exist but for children they are unsuitable introductions to the concept of angle. Hans Freudenthal [9, p.444] cites the enormous trouble

required to construct angles using linear algebra and the fact that it does not yield proof of all geometrical results in any case. By contrast the static approach enables angles to be introduced very quickly but does not offer experience of them as rotations, their one aspect which distinguishes them from other concepts met at school, such as length, area or sets. In fact many advanced and school text books which initially adopt static angle definitions attempt to overcome the conceptual as well as the mathematical limitations by referring to angles as rotations at a later stage. Heath [2, p.179] states that this tendency indicates that 'the essential nature of an angle is closely connected with rotation'.

Rules for measurement of angles cannot be introduced until conventions regarding sense (orientation) and multiple turns have been discussed. Consequently, for school pupils, an approach integrating both measurement and concept development but excluding axiomatic foundations is required. The visual aspect of the static angle and motion involved in the dynamic approach are both important in developing a concept of angle.

Freudenthal [9, pp 476-494] details three main concepts of angle, each of which is important didactically and can be developed at various levels of rigour. He considers these concepts and their limitations and applications rather than any historical or axiomatic comparisons. He includes both static and dynamic definitions of angle and stresses the need for the parallel introduction to angle concepts and measurement as the latter offers concrete experiences which assist

in developing the former. The concepts are described below and arise from:

1. Elementary geometry,
2. Goniometry (trigonometry),
3. Analytic geometry.

In elementary geometry angle is defined in terms of a non-ordered pair of half-lines in the non-oriented plane and determined between 0° and 180°. Euclid's angle concept fell into this category yet he still created larger angles as abstract sums of the visualisable angles between 0° and 180°. Freudenthal compares this with adding the lengths of line segments that are not on the same line or with adding weights of eggs, the union of which is clearly not an egg. Subtraction cannot be performed as the addition of a negative magnitude since no negative magnitudes exist.

In goniometry angle is defined in terms of an ordered pair of half-lines in the oriented plane and determined mod 2π. The angle is viewed as a centre angle of a circle of unit radius such that arcs and angles correspond to each other. The use of radian measure follows naturally. Orientation of the plane precedes angle definition and angles greater than 2π are only representable mod 2π. Both addition and subtraction can be performed mod 2π and interpreted functionally as rotations and reflections.

In analytic geometry angle is defined in terms of an ordered pair of lines in the oriented plane and determined mod π. If sine and cosine are considered as analytic functions they are defined for any real argument and are periodic mod 2π. If the arguments are considered as angles

then angles must be allowed from $-\infty$ to ∞. Freudenthal calls these analytic or turn around angles. Their introduction assists with the goniometric concept by comparison and offers a means to concretize all sums of elementary geometry angles beyond 180°.

Freudenthal distinguishes the three angle concepts with the terminology 'absolute angles', 'angles' and 'turn around angles'. He states that they can be concretized by the measuring process using respectively a semicircle (half) protractor, a full circle protractor and a hodometer or electricity meter which registers whole and part turns.

The above notions which Freudenthal describes can be developed at an elementary experiential level and, because examples of each are met by children in everyday life, none should be glossed over in reaching an understanding of angle and the conventions relating to it. Children's earliest experiences of angle must be both static and dynamic. They should meet static angles in as many contexts as possible and recognise them in drawings of ray pairs and corners. They should also experience angle as a rotation through multiple and part turns which, in contrast to static angles, will often not be recorded diagrammatically. Appropriate measuring instruments should be introduced to reinforce the angle concepts.

Realistic goals for first year secondary school pupils (age 12) are the ability to compare and to measure angles.

CHAPTER 3

CHILDREN'S UNDERSTANDING OF ANGLE - A LITERATURE REVIEW

The reasons for undertaking a literature review are outlined in the introduction. There is only a small amount of literature relating to children's understanding of angle. It is reviewed in the second section where the major misconceptions revealed by research are described. The third section offers a brief critical appraisal and summary of these findings.

3.1 INTRODUCTION

A literature review was undertaken in order to highlight the areas of angle in which the most serious misconceptions arose. Because its purpose was only to suggest misconceptions which might or might not obtain locally, it took the form of a qualitative analysis of related research.

The most serious misconceptions revealed would be used as a guideline for the construction of items in the primary tests which themselves were intended to elicit preliminary local information from which the detailed secondary tests would be drawn up.

3.2 LITERATURE REVIEW

G.E.T. Holloway [10] has pointed to the reliance which young children place on intuition rather than measurement in the comparison of angles and to their confusion regarding which aspect of the angle, such as arm length, should be measured. His findings highlight children's difficulties in relating the act and result of measuring to the attribute which is being measured. Although much of his work is with seven year olds these problems remain,for many pupils, well into secondary school and are borne out with eleven year olds in the APU Primary Survey (see page 25).

Geoff Giles [11, p.153], in his research has found that 'the concepts involved in 'angle' are not easy for children to grasp'. The comparative size of angles was often confused with the size of the diagrams (i.e. arm length) representing the angles. Approximately half of his 4881 subjects made this sort of error in listing five angles in order of size. Many pupils experienced difficulty in identifying all angles in diagrams with more than two rays emanating from a vertex and over half, when shown such diagrams, indicated that they were not sure that an angle had exactly two arms.

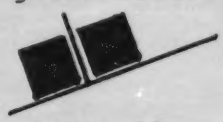
Hartmut Wallrabenstein [12] found from his research with nine to eleven year olds that three quarters of the children connected wrong ideas with certain technical terms such as 'parallel', 'direction' and 'distance'. He pointed out the need for minimising the use of non-standard terminology and of abstract diagrams in test items. In teaching, however, the most appropriate time for introducing a technical

term to replace unspecific paraphrases must be selected.

Wallrabenstein noted the great difficulties which children experienced in distinguishing between right and left turns except when made from their own standing place. This corresponds with observations of the difficulties which even older children experience in distinguishing between clockwise and anti-clockwise rotations.

Wallrabenstein expressed surprise in finding that, although parallel lines (exhibited as edges of a strip) were recognised in various situations, right angles were often not recognised. He developed a test item in which a square plate was illustrated as fitting into a right angled corner between two rods and asked subjects to draw a copy of the plate into all corners where it fitted precisely in other diagrams of rods. The item is reproduced below and was very poorly answered by his subjects.

'This plate fits into the corner between the green rods.



Draw a plate into all corners where it fits precisely.

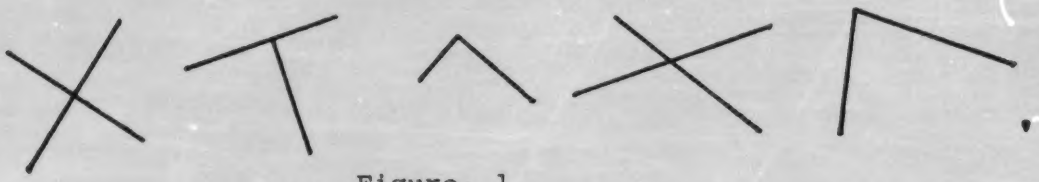


Figure 1

The problem of the sequencing of the naming and recognising of shapes, also experienced with right angles, is frequently debated. Both the Schools Council in their

'Early Mathematical Experiences' [13] series and 'Nuffield Mathematics Project' [14] encourage the early use of the correct names for shapes encountered. The D.E.S. in 'Mathematics 5-11 A Handbook of Suggestions' [15] advocate the development of an awareness of shapes and their attributes prior to naming. Jenny Murray [16], in supporting the D.E.S. view, comments that the premature imposition of vocabulary can cause investigation and thinking regarding the properties involved to cease. It can be seen that the ability or inability to name, say, a right angle, can be unrelated to whether the pupil possesses the concept of a right angle, and is greatly affected by the approach adopted in their primary school.

David Fielker [17] has also observed problems relating to right angles. He noted the confusion between the words rectangle and right angle. He found that some bright pupils were only able to halve a right angle when told that it was 'an angle of 90 degrees'. From this he inferred that, 'the right angle for them was not a particular case of a more general angle', and he attributed this to either 'the strong feelings for perpendicularity that all children seem to have', or the 'teaching of square corners' before a more general idea of angle [17, p.108]. The APU results also (see page 24) reveal the difficulties many pupils experienced in halving right angles.

The widest ranging relevant research reports are those of the D.E.S. Assessment of Performance Unit (APU), Primary Surveys the second of which was undertaken with 14,500 eleven year old pupils. For details of the 1979 results of the

practical tests on angles, taken by 1000 pupils, see Report Number 2 [18, pp 19-24].

It was found that only 55% of pupils were able to name a right angle made from two hinged strips yet 60% were able to state correctly the number of degrees in a right angle. This corresponded to results of the written test item in which pupils were presented with a drawing of a right angle, which was not named as such, and asked to estimate its size. The reporters remark [18, p.20]:

> 'Evidently some pupils who had forgotten the name of the angle remembered its properties when the name was supplied.'

When one hinged strip was rotated through a full turn 80% of the pupils stated correctly that it had turned through four right angles but only 60% were able to convert this answer into the correct number of degrees.

Over 80% of pupils were able to fold an irregular sheet of paper twice to make a right angle but fewer were able to identify the position of the angle on the folded piece of paper and consequently some folded the paper in half again in a variety of incorrect ways when asked to halve their right angle. They seemed to have perceived a right angle as the whole piece of paper and perhaps as an area.

For the measuring items pupils were allowed to select a half protractor, full static protractor or rotating 'Truefitt' angle indicator. Most pupils knew that these were used to measure angles but approximately 60% were unable to name

them correctly. The selection of the half protractor by 80% of the pupils reflected its widespread use in primary schools. The four angles to be measured consisted of three acute and one obtuse, all with one horizontal arm. Testers taught pupils how to use the protractor if they failed on the first item. The facilities for each task were respectively 40%, 47%, 55% and 55% with fewer than 5% of pupils reading the wrong scale for acute angles but 20% doing so for the obtuse angle. The reporters comment that this shows that certain pupils are proficient in using the protractor but lack understanding of the relative size of angles. They also cite the written test results in which angle estimation, especially of obtuse angles, was poorly attempted.

In conclusion the testers noted pupils' lack of understanding of the nature of angles and commented [18, p.23]:

> 'The pupil is very proficient at measuring angles but her responses to probes seem to indicate that she does not know what she is measuring, i.e. doesn't really know what an angle is.'.

3.3 CRITICAL APPRAISAL AND SUMMARY

From the general comments of both Giles and the APU it is clear that children find angle hard to understand. Holloway and the APU have both noted the disparity between proficiency in measurement and understanding of angles' attributes. Wallrabenstein, Fielker and the APU have revealed difficulties in recognising and understanding the nature of a right angle. Giles has noted that arm length is incorrectly

considered to affect angle size and that problems occur when there are more than two angles at a vertex. Wallrabenstein has stressed the need for minimising technical terms and abstract diagrams and has pointed to the confusion between senses of rotation. The APU has commented on the difficulty of estimating angle size, the errors arising in angle measurement and on the widespread use of the half protractor in primary schools.

It was felt that the most serious misconceptions were those relating to the attributes affecting the size of an angle and that these were followed closely by those relating to the right angle. It was noted that some of the research results were questionable but they were quoted as they were only intended as rough guidelines.

Some responses could have been affected by the mode of presentation of the items, some inferences were based on small amounts of evidence and the APU policy of teaching while testing could result in the testing of only immediate recall.

CHAPTER 4

THE PRIMARY TEST AND INTERVIEWS

The purpose of the primary test and interviews is outlined in the first section while the second and third sections describe the subjects selected and the details of administration. The sources of item areas and their allocation to either the test or the interview are discussed in section four. In the fifth section each item in the test and the responses to it are analysed and recommendations are given for secondary test items. In the sixth section the interviews are described, each interview is analysed in detail and the resulting insights are discussed.

4.1 PURPOSE

The primary test was intended to highlight areas of misunderstanding while the interviews were intended to probe the causes of these more deeply. The interviews would also be used to reveal problems relating to measurement and to provide a means for testing additional items which could not be included easily in a written test. The findings from the test and interviews, together with the results of the analysis of primary texts, would be used in making recommendations for the improvement of the presentation of angle in primary school and in formulating the more detailed secondary test and interviews.

4.2 SELECTION OF SUBJECTS

The primary tests and interviews were carried out with fourth year primary school pupils (aged 11) during the term before they were due to transfer to my secondary school. The pupils were selected from the four main feeder primary schools which together provided a total of 80 of our first year intake of 130, the remainder coming from a further 22 schools.

These pupils were of all abilities but the proportion of high ability ones was far smaller than the proportion of high ability pupils among all eleven year olds in the country (as shown by nationally validated tests administered by primary schools).

4.3 ADMINISTRATION

After obtaining the permission of the head teacher the appropriate number of tests were issued to the teacher in charge of mathematics at each of the four schools. Only one school administered the test. It was taken by 18 of our prospective pupils who came from three separate classes.

The test was presented on two A4 pages handwritten using two colours of banda. It appears in its original format on page 13 of the Appendix. The arcs at each vertex were drawn in red, as were the dotted lines on which pupils were to answer the questions.

The pupils did not sit the test in examination conditions but the exact mode of its presentation was not known. It was not intended that assistance should be given or that interpretation queries should need to be made. The nature and

presentation of the items had been chosen accordingly.

In each of the four schools three pupils (1 of high ability, 1 of average ability and 1 of low ability) were interviewed. These interviews were tape recorded but, due to problems with the recorder, some of them were lost. The remaining ones were transcribed for later analysis.

For testing proficiency in measuring, a sheet of angle drawings (see Appendix,page 16) was used. A half protractor and rotatable full protractor, the 'Truefitt' angle indicator, were provided. These are both shown in Figure 2 on page 48.

4.4 SELECTION OF ITEM AREAS

In choosing the items to be included in the primary test and interviews it was necessary to concentrate on certain areas suggested by the educational research findings of Chapter 3 while also considering the recommendations of section 2.3, pedagogical implications. Errors previously noted in my school regarding the recognition of right angles in various orientations and the sense of a rotation were also taken into account. For each item area the most appropriate format, written or verbal, was chosen. All written items were also used in the interview. The item areas selected are described below.

The lack of correlation between understanding of the angle concept and proficiency in measuring suggested that these aspects should be tested separately. The former could be administered in a written format while the latter required

an interactive setting if incorrect measuring procedures were to be observed. It was also impossible to provide sufficient protractors for use in a large scale written test. Consequently both the primary test and interviews included items on the concept of angle while the interviews also included items on measurement.

It was felt that the most serious misconceptions were those related to the attributes of angle deemed important in affecting its size, such as arm length and positioning on the page (including orientation). Consequently many items requiring visual size comparison were constructed. A few reflex angles were also included to test pupils' ability to extend their angle concept.

It is important that pupils are able to recognise right angles, which are a central angle concept both historically and mathematically. Accordingly right angles were shown in both familiar and unusual orientations in different types of item. In order to minimise the use of technical terms and the effects due to primary school policy in introducing them it was decided not to construct items utilising terminology such as 'acute' or 'obtuse' but, due to its importance, to include one item on the naming of a right angle. In order to include both static and dynamic approaches to angle items relating to the number of degrees in a right angle and in various turns were written.

Items requiring the estimation of the number of degrees in acute, obtuse and reflex angles were constructed, although many pupils had not previously met reflex angles in this form (see p.87).

In order to ascertain the extent of brighter pupils' experience two items involving triangles and one item with many angles at each vertex were included.

The test included the following item areas: size comparison of acute, obtuse and reflex angles; recognition of equal angles, including right angles, in various orientations; the naming of a right angle; stating the number of degrees in a half turn, full turn and right angle; approximating angle sizes in degrees; the angle sum of a triangle; recognition of equal angles between intersecting pairs of parallel lines.

In the interviews angles of various sizes in different orientations were measured using both half and full rotatable protractors. Questions regarding the nature of an angle, the sense of a rotation, the meaning of 'acute' and 'obtuse' and examples of right angles in the room were asked. The items in the written test were also worked through.

4.5 THE TEST

Each item is first discussed separately and then the interpretations are summarised and recommendations made for adaptations of the items for use in the secondary test.

4.5.1 THE TEST ITEMS - OBJECTIVES, DESCRIPTION, RESULTS AND INTERPRETATION

Each item is reproduced together with the correct answer. The objectives tested and details of presentation are given. Noteworthy responses are outlined and interpretations made.

The responses of each pupil to each item and the percentage facility for each item are shown in Table 1 on page 43 .

The full test, in its original format, is in the Appendix on page 13.

Item 1

Which is the LARGEST of these angles? _____

(a) (b) (c) (d)

Answer: (b)

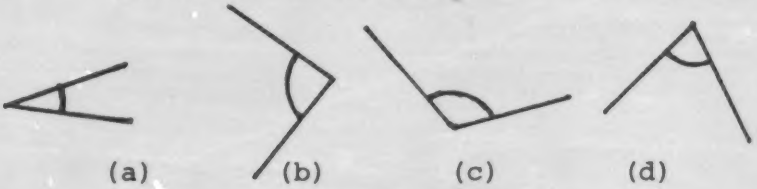
This item was presented to find out if pupils possessed the most elementary notions of angle. It does not include obtuse or reflex angles, differing length arms or angles with two oblique arms.

All arms of these angles are the same length. The left arm is placed vertically, a direction from which many angles in text books are measured. The right angle is not marked with the customary sign as the intention of the item is only to test visual comparison of angle size.

All eighteen pupils answered this item correctly.

Item 2

Which is the SMALLEST of these angles? _____



(a) (b) (c) (d)

Answer: (a)

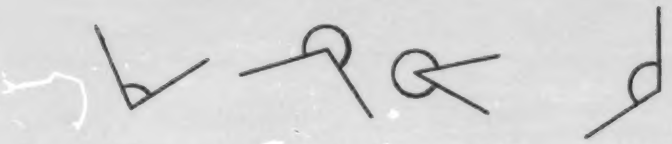
This item tests the notion of angle size, independent of orientation, for acute and obtuse angles. In so doing it includes two more concepts (obtuse angles and various orientations) than item 1.

All arms of these angles are the same length. The angles are either acute or obtuse.

Only two pupils answered this item incorrectly, choosing (c), a likely error if the question was misread as asking for the largest angle, the question posed in item 1.

Item 3

Which is the LARGEST of these angles? _____



(a) (b) (c) (d)

Answer: (c)

This item tests the notion of angle size for acute, obtuse and reflex angles.

All arms of these angles are the same length. There is one each of acute, obtuse, reflex less than 270° and reflex greater than 270° angles.

Nine pupils answered this correctly but (b) and (d), angles with arms spread wider apart than those of (a) and (c), were common errors. This indicated a static perception of angle as the sharpness of a point, an idea which could be applied successfully to acute and obtuse angles but not to reflex angles. Using this approach the red arc marked at the vertex would necessarily be overlooked as its purpose in the case of reflex angles could not be understood.

Item 4

What is the special name for this angle?

Answer: a right angle

This item tests the combination of recognition of and recall of the name of a right angle.

The angle is drawn with its arms parallel to the sides of the page so that it is shown in a familiar orientation. The customary symbol for the right angle is included as an extra cue because it is often introduced to pupils at an early stage.

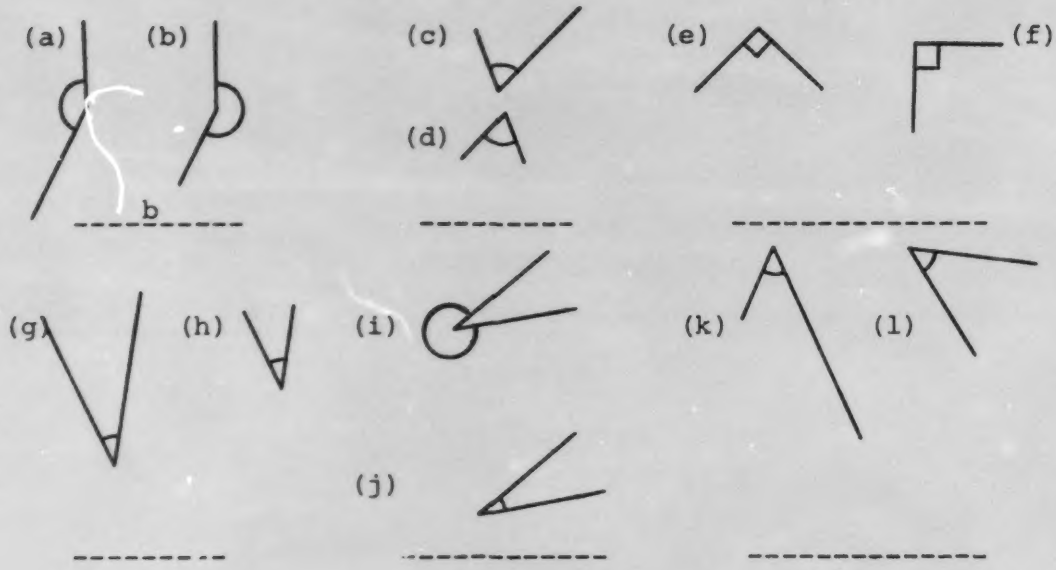
It is important to include items on the right angle in the written test and recognition is far more important than the mere naming of a right angle (see discussion on page 22). It was, however, thought to be difficult to phrase a question on the recognition of right angles in the room when the test was to be administered by unknown primary teachers. Consequently this item on naming a right angle appears alone, without other interpretative items.

It was expected that responses to this item would not indicate the number of pupils who understood the concept of a right angle.

Ten pupils named the angle correctly but seven pupils did not even attempt the item.

Item 5

Which angle is BIGGER or are they the SAME size?
Answer this question for each pair of angles.



Answers:

(c) (d)	(e) (f)	(g) (h)	(i) (j)	(k) (l)
same	same	same	(i)	same

The parts of this item are constructed to test the attributes of an angle deemed important in affecting its size. Those tested are: the length of one or both arms, orientation and relative position.

The provision of an answer to the first part is intended not only to provide a model for the mode of answering but also to exhibit an example including a reflex angle. The wording of the question is already long enough without instructions on the mode of answering.

From the example pupils inferred that the letter of the larger angle was to be entered on the dotted line. They also seemed anxious to find most angle pairs different, rather than

the same, although four of the five pairs given were the same size. The angles were carefully drawn but the pupils had no way of exact comparison by tracing or measuring and therefore judged equal angles as unequal because they felt them to be slightly different in size.

Angles (c) and (d) are equal and drawn as slightly separated vertically opposite angles because these often look intuitively equal. Both arms of (d) are equal in length to the shorter arm of (c). Pupils who feel that arm length affects the size of an angle will opt for (c) as the larger. Four pupils answered correctly while six selected (c) and six selected (d).

Angles (e) and (f) are right angles and are presented to ascertain whether or not a right angle in an unusual orientation (e) is recognised. Angle (f) is also drawn in a different orientation from that of the right angle in item 4. The right angle symbol is used to give extra assistance to those who recognise it. Ten pupils answered correctly while two selected (e) and five selected (f).

Angles (g) and (h) are equal and oriented identically although not with their arms placed symmetrically about the vertical. They each have equal length arms and are positioned in such a way that their angles can easily be seen to be equal. The vertex of angle (h) is above that of angle (g). This part tests whether an angle with longer arms or an angle with its vertex higher on the page is judged to be larger. Seven pupils answered correctly and eight chose (g) as larger. Of these eight pupils, four had selected (c) as larger than (d) while three had selected (d) as larger than (c). Four

pupils had therefore consistently selected the angle with a longer arm or longer arms as larger.

Angles (i) and (j) are oriented identically with arms equal in length to each other and with (i) above (j). Their sum is 360^{o}. It is intended that the most important variable to be considered is the rotation represented by the red arc although position is also a variable. This part is intended to test whether the reflex angle between two rays is recognised as larger than the acute angle between the same two rays. Twelve pupils answered correctly and four stated that the angles were equal. The high facility may be due to the similarity with the worked example or to the nature of the item.

Angles (k) and (l), are equal and oriented differently from each other but with their vertices above their arms, which is a fairly unfamiliar presentation. The arms of (l) are equal while one arm of (k) is longer and one is shorter than those of (l). It was intended to test whether an angle with equal fairly long arms is judged to be larger or smaller than an angle with one very long arm. Seven pupils answered correctly while five selected (k) as larger and three selected (l) as larger.

Item 6

How many degrees are there in ONE FULL TURN? _____

Answer: 360

Item 7

How many degrees are there in a HALF TURN? _____

Answer: 180

These items test pupils' familiarity with degree measure in a dynamic context.

Four pupils answered item 6 correctly of whom three were the only ones to answer item 7 correctly. Four pupils suggested 180 as an answer for item 6 of whom three also suggested 90 for item 7. Two of the pupils who answered 180 for item 6 and 90 for item 7 went on to state that there were 90° in a right angle, apparently without noticing their inconsistency.

It seems that some pupils have met 180° in discussions about rotations or on their half protractor and think that it is the largest possible angle. A straight angle (half turn) can easily be misinterpreted as a full turn because its arms lie on the same line, the line indicating the direction from which the turn was started. It is only the position of the arc which distinguishes the drawing of the full turn from that of the half turn, as illustrated in Figure 3. There is also a linguistic confusion because, after making a half turn one

A full turn A half turn

Figure 3

has 'turned round' to face in the opposite direction. The general usage of 'turn round' requests only a half turn to be made although the wording seems to be referring to a full turn.

Item 8

How many degrees are there in this angle?

Answer: 90

The item is presented in this format to ascertain pupils' familiarity with the degree measure of a right angle viewed as a static angle rather than as a quarter turn.

This drawing of a right angle uses an orientation different from the two used earlier in the test. It also uses the right angle symbol.

Five pupils answered this item correctly of whom three were those who had answered both items 6 and 7 correctly.

Two pupils answered this item with 2 and one pupil with 1 indicating that the unit of right angle and half right angle, introduced prior to degrees in many texts, had remained in their minds as the 'way' in which to measure angles. Unfortunately each one of these pupils had recalled these units incorrectly as is shown by the inconsistency of their use in responses to other items.

Item 9

About how many degrees are there in each of these angles?

--- --- --- --- ---

Answers:	45	78	158	267	343
Acceptable:	40-50	70-85	150-170	260-280	330-355

This item is intended to test how well pupils can apply their knowledge of degree measure.

Only seven pupils gave any acceptable responses in the whole item of whom four were only able to estimate the size of the first angle correctly. This is a half right angle, which is frequently met in early work on angle. Altogether there were six correct responses to the first item and then one or two to each of the others. Only three pupils answered correctly any part other than the first.

Item 10

What do the 3 angles of a triangle add up to? ------

Answer: 180°

This item is intended to reveal whether the angle sum of a triangle has been met and, if so, if it has been recalled.

Only five pupils were able to attempt this item, none of them correctly.

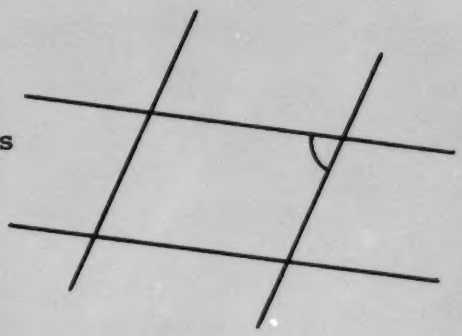
There were very few pupils who knew that there were 180° in a half turn and so, even if they were able to recall that the three corners of a triangle could be torn off and formed into a straight angle, they would not realise that this could be expressed as 180°. The wording, 'What do the 3 angles of a triangle make?' could have prompted the response, 'a straight angle' even though the question is phrased rather unspecifically.

The abstract nature of the question rendered it ineffective in revealing the number of pupils who had

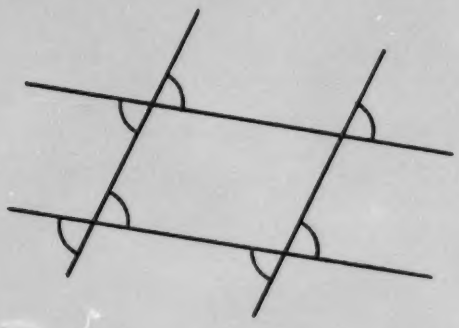
previously experienced any explanation of why the angles of a triangle could form a straight angle.

Item 11

Mark with a curve 2 other angles <u>equal</u> to the marked one.



Answer: two of

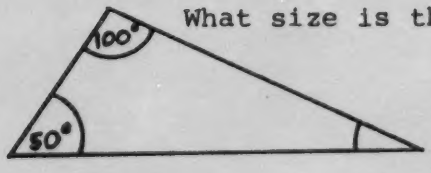


This item is intended to test the ability to recognise more than one angle at a vertex and to identify equal angles in a diagram including a large number of them.

Five pupils answered this item correctly and five omitted it.

The diagram was badly drawn with insufficient difference in size between the acute and obtuse angle at each vertex. This doubtless contributed to the low facility. The item did not ask pupils for angles which were 'about equal' nor did it explain that the pairs of lines were parallel, yet it presupposed notions about corresponding and alternate angles.

Item 12



What size is the missing angle in this triangle? ------

Answer: 30^{o}

This item tests the application of the knowledge that the angle sum of a triangle is 180°.

The values 100° and 50° are chosen to minimise arithmetical errors and to alleviate the need to translate the question into a subtraction, a difficulty discussed in K.M. Hart (ed) [19, page 29].

In the drawing the actual size of the angle is 25°. It was obviously measured correctly by three pupils. The two pupils who gave the correct response, 30°, had been unable to answer any of the other items involving degrees correctly and therefore had measured incorrectly. It is not clear whether the five who answered 150° had measured inaccurately using the wrong scale or had merely added the two given angles because they recalled only part of the algorithm. Five pupils omitted the item.

4.5.2 DISCUSSION OF INTERPRETATIONS AND RECOMMENDATIONS FOR SECONDARY TEST ITEMS

In item 1 success could be due to observing the time of the clock position of the right arm rather than the appreciation that (b) represented the largest rotation. This interpretation of angle could be the result of mistaking it for a fixed positional measure such as a bearing. The correct responses to item 2, where acute and obtuse angles were differently oriented, indicate that this was not the case. The success on both items suggests that the easier, item 1, can be omitted in the secondary test.

In view of the reading errors in item 2 it was felt preferable to restrict items in the secondary test to the

TABLE 1

PRIMARY TEST RESULTS

ITEM NUMBER

	1	2	3	4	5i	5ii	5iv	5v	5vi	6	7	8	9i	9ii	9iii	9iv	9v	10	11	12	
CORRECT RESPONSE	b	a	c	✓	s	s	s	i	s	360	180	90	+5	78	158	267	343	180	✓	30	20

PUPILS

																					PUPIL'S SCORE (mean 6·9)
Edith	b	c	a	✓	c	f	g	i	l	100	50	80	20	40	60	70	10	120	✓	30	5
Jenny	b	a	b		d	s	g	i	s										X		5
Colin	b	a	b	✓	d	f	s	i	l	360	180	90	50	80	110	270	358		✓	150	12
Vanessa	b	a	c	✓	s	s	s	i	s	180	90	90	40	170	160	270	30		✓	25	14
Petrula	b	c	a	✓	b	s	h	s	h	180	90	90	70	30				25	X	25	4
Kerry	b	a	c		d	s	g	i	k	180	90		50	60	90	90	30			80	6
Alan	b	a	c		s	s	s	i	s	180	60	5	5	10	15	20	30		✓	30	10
Mustapha	b	a	d		d	e	s	s	k	3	2	1	1	2		3		5		150	3
Taufiq	b	a	c	✓	s	e	g	i	s	100	50	60	50	8	100	9	10		X	25	8
Diego	b	a	c		c	f	g	i	c												4
Susana	b	a	c		s	s	s		s				10	20	50	100	100		X		7
Laura	b	a	d	✓	c	s	s	i	s	1	¼	2	1	2	4	3	5				7
Robert	b	a	c	✓	d	s	g	i	l	360	180	90	40	50	65	55	10	270	X	150	10
Bernadette	b	a	d	✓	c	f	g	i	k										X	20	4
Perry	b	a	c	✓	d	s	s	i	s	360	180	90	70	80	360	330	350		X	90	13
Steven	b	a	d	✓	c	f	h	s	k	3	2	2	1	2	4	3	1	5	✓	150	4
Rajan	b	a	b	x	c	s	g	s	k	360	150	130	50	20	20	60	40		X	150	5
Bridgit	b	a	c																		3

18

NUMBER OF ACCEPTABLE RESPONSES	18	16	9	10	4	10	7	12	7	4	3	5	6	2	1	2	1	0	5	2	124
PERCENTAGE FACILITY	100	89	50	56	22	56	39	67	39	22	17	28	33	11	6	11	6	0	28	11	

Key

- s represents the response 'SAME'
- ✓ represents a correct response
- X represents an incorrect response
- represents an omission

Notes

The ranges of acceptable responses for item 9 are:

9i	40-50
9ii	70-85
9iii	150-170
9iv	260-280
9v	330-355

selection of only the largest angle. This would still enable items to be constructed in which the answers were acute, obtuse and reflex angles. The results showed that pupils able to distinguish the largest angle in a group were also able to distinguish the smallest. Therefore the latter notion did not need to be tested in the secondary test.

The large drop in facility between the first two and the third items indicated the lack of familiarity with reflex ray pair angles.

It was felt that the small number of correct responses to item 4 did not reflect the number of pupils able to recognise a right angle. Consequently it was considered important that an item on recognition of a right angle be included in the secondary test. This would be possible as the test would be administered by staff who could be briefed beforehand.

The problem of pupils judging equal angles in item 5 as unequal because they felt that one angle was slightly bigger than the other could be alleviated by the rewording of the question in the secondary test. An improved format would be: 'Which angle is BIGGER or are they ABOUT THE SAME SIZE?'

More information was required to aid interpretation of the large number of pupils selecting angle (d) as larger than (c).

It was felt that the inclusion of the right angle symbol in (e) and (f) complicated the interpretation of whether correct results were due to the recognition of the equality of the angle size or of the symbol. Omission of the symbol

would remove this complication.

It was not clear whether (i) was selected as larger than (j) for the correct reasons, because (i) was above (j) or because of the similarity to the worked example. It was thought that the removal of the similar worked example would assist in interpretation of the responses while its loss could be compensated for by the administration of the test by well briefed teachers.

The arms of (l) were not sufficiently longer than the shorter arm of (k) and, due to their orientation, may have been assumed to be equal to it. The choice of (k) as larger would thus have been more natural than the choice of (l). Such difficulties in comparing the lengths of differently oriented lines are pointed out by K.M. Hart [19, page 22] who found that about 50% of secondary school pupils tested were not convinced that the line segments which form a side and a diagonal of a rectangle are different in length. In order to test the comparison of size of two equal angles one of which has equal medium length arms and the other of which has one arm longer and one shorter than this, each arm length must be visibly very different.

The poor responses to items 6 and 7 and the fact that they were answered correctly by mostly the same pupils suggested that one of the items was superfluous. As 180° was a frequent incorrect answer for a variety of items, it would be more revealing to retain the item asking for the number of degrees in one full turn. It seemed that the order of presentation of this type of item could be improved. If the first such item asked for the degree measure of a right

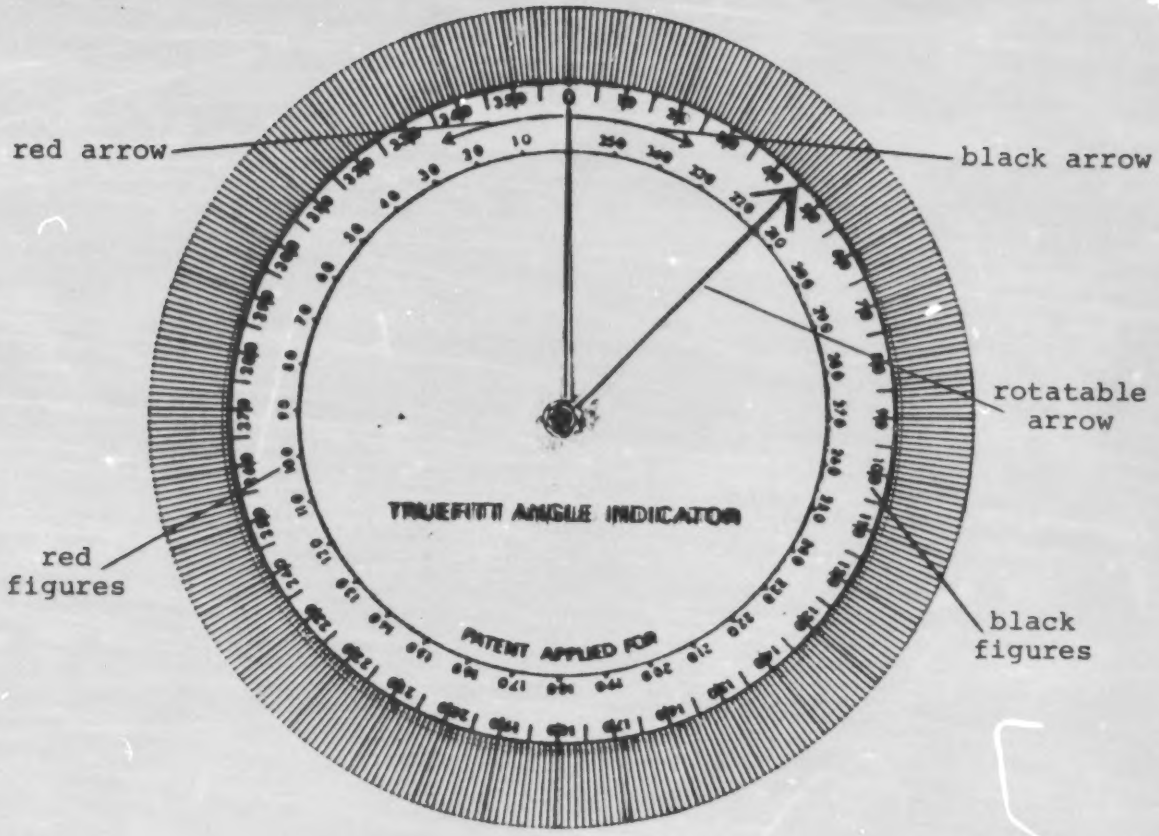
angle, which is the most widely taught, the degree measure of larger turns could be calculated as multiples of 90°. This is the order used by the APU in their primary tests [18, page 20] but they still find that the inability to calculate multiples of 90° results in correct answers from fewer pupils than those who can express the same turns as the correct number of right angles.

During the analysis of item 9 it became clear that this was too hard for many pupils in spite of the fact that they had previously been taught to measure using half protractors and had consequently been given the opportunity to relate angle measures to angle sizes. There were many wildly inaccurate answers, even from those pupils who had correctly stated the number of degrees in a right angle or in full and half turns. These responses were the first signs in the practical research of the inability of pupils to relate the skill of measurement to the size of an angle, a problem discussed in section 3.2 and pointed out by Holloway [10] and the APU [18].

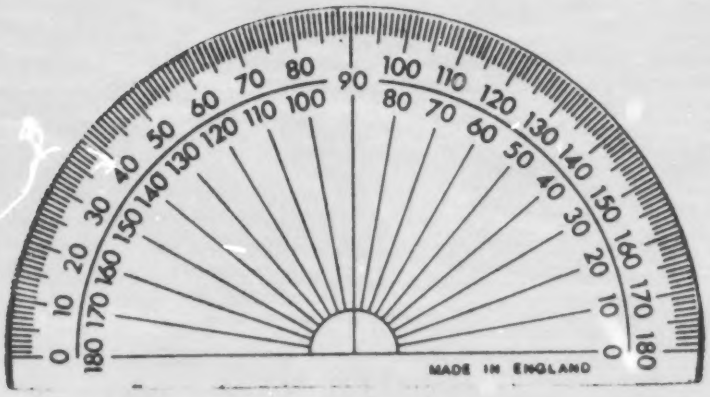
Responses to items 10 and 12 revealed that none of the pupils were familiar with the angle sum of a triangle. The number of prerequisite concepts lacking in most pupils made clear the need to alleviate these well before the introduction of angles in a triangle. It was therefore felt that items relating to angles in triangles should be omitted from the secondary test and that the topic should be introduced from the most basic level once the more elementary misconceptions had been overcome by presentation of the angles unit.

The complex nature of item 11 rendered it unsuitable for inclusion in the secondary test.

The large number of basic misconceptions brought to light by the primary test alone suggested sufficient material for use in the secondary test. Unless these serious misconceptions were probed and then eradicated, no further angle based concepts would be understood. It was therefore decided that items other than the most basic should not appear in the secondary test.



FULL ROTATABLE PROTRACTOR



HALF PROTRACTOR

Figure 2

4.6 THE INTERVIEWS

The content of the interviews is described, each interview is analysed and summarised and finally the overall insights gained are discussed.

4.6.1 CONTENT

Each pupil was asked questions from the primary angles test (Appendix, page 13) and was asked to explain the reasoning behind some of their responses.

A sheet of drawings of acute, obtuse and reflex angles (Appendix, page 16) in various orientations and with different arm lengths was used to test proficiency in measuring. Both half and full rotatable protractors were used by each pupil. These are illustrated in Figure 2 on page 48 . The 'Truefitt' angle indicator is numbered clockwise in black and anti-clockwise in red. A black and a red arrow indicate the colour of scale to be read for each sense of rotation. The arrow along a radius of the indicator is drawn on a separate piece of acetate from the scales and can be rotated freely about the centre of the indicator.

Four additional questions were posed. Pupils were asked to identify examples of right angles in the classroom. They were shown examples of acute and obtuse angles and asked if they could recall the words used to describe them. They were asked whether angles were used to measure lengths, areas or amounts of turning. Some were also asked whether the clockwise and anticlockwise measurement of an angle gave the same answer.

4.6.2 ANALYSIS OF INTERVIEW TRANSCRIPTS

All seven remaining taped interviews were transcribed in note form. Presented here are the main responses given by each pupil combined with an interpretation of these.

After this detailed analysis of each interview a brief impression of the pupil's abilities and misconceptions relating to angle is given.

B I M A L

Bimal was able to recognise the largest and smallest of the acute and obtuse angles but had not met reflex angles before so selected the obtuse angle as largest.

He could not name the right angle and when asked to print out some right angles appeared to recognise the word and then selected the two corners on the right of the question paper. He momentarily thought that a corner on the left of the paper was also a right angle but withdrew this correct answer quickly, apparently realising that it was on the left of the paper.

He selected 5(c), 5(g) and 5(k) as the larger of their pair explaining that this was because they had longer arms. When asked if there was any special name for 5(e) and 5(f) he made no response. He could not see that either of them were right angles, further emphasising his misconception that right angles were on the right.

5(i) was recognised as larger than 5(j) 'because the pink is longer', in other words because the pink arc was longer.

He was unable to attempt items 6 or 7 but recognised the right angle in item 8, which pointed towards the right, as ninety degrees.

His estimate for the first part in item 9 was 100, which was clearly a wild guess. He was unable to attempt the items about a triangle and guessed many equal angles, some correctly and some incorrectly, in item 11.

He measured angle 5(g) with a half protractor as 105° and 5(h) as 105°. They are both in fact 33°. His answer of 105° was an attempt to read 150° which was the value to the nearest ten on the wrong scale. After measuring both 5(g) and 5(h) he still did not see the connection between their equal measures and their relative size. When it was pointed out to him that he had said 5(g) was larger than 5(h) but that they both measured 105° the inconsistency was not apparent, perhaps because he did not know what aspect of the angle the protractor was measuring or the relative importance of this measure and of the arm length in determining the size of an angle.

Bimal was only asked to measure angle A from the sheet of angle drawings with both protractors. Due to his low level of motor skills, his answers were both slightly inaccurate, although read from the correct scale. He said that he found it easier to measure the angle with the full protractor than the half protractor.

Summary

Bimal is a very weak pupil, who has heard the words 'right angle' and 'ninety degrees' but is unsure of their meaning. His criteria for comparing angle size are sometimes

the sharpness of the point indicated by the length of the red arc and at other times the length of the arms. He has not met reflex angles nor is he able to fit them into his schema at present. He is also unsure of the purpose of measuring and lacking in the required skills and the ability to interpret his results.

R I F A I T

Rifait answered items 1 and 2 correctly but chose (d), the obtuse angle in item 3. She had not met reflex angles before. She could not give the words 'acute' or 'right angle' but stated that she recognised them. After 20 seconds and prompting she said that there were right angles in the room and window corners.

She found 5(c) larger than 5(d). When asked if it was slightly larger or much larger she stated that it was much larger because 5(c) had a longer arm. 5(e) and 5(f) were said to be, 'not exactly the same', indicating that she could not recognise them both as right angles. Her reason for selecting 5(g) was that it was 'much larger' and for 5(i) that its arms were 'more spread apart', an unclear point which she did not amplify. The arms of both 5(i) and 5(j) are in fact equally far apart. The use of the word 'spread' implies that she was focussing attention upon their static relative position rather than the length of the red arc indicating the amount of rotation. The reason for her reply is not therefore apparent.

She found 5(k) larger than 5(l) because 'it has one big arm and one small arm'. It is not clear why the small

arm is mentioned as the selection of 5(k) as larger appears to be due to the longer arm of 5(k) being longer than the arms of 5(l). She has either not noticed or found it unimportant that the arms of 5(l) are both longer than the short arm of 5(k) although her comment '... one small arm' may refer to the comparative size of the arms of 5(k) or of the small arm of 5(k) with those of 5(l). It is hard to follow up these vague comments with pupils who are weak at verbal expression. It is also probable that her criteria for comparing angles, if they could be ascertained, are not fixed or logical.

She was given a half protractor and measured both 5(g) and 5(h) incorrectly as 70° by approximately bisecting the angle with the 90° line. She concluded from the results that 5(g) and 5(h) were the same size. When asked if the length of the arms made 5(g) bigger she replied 'no miss - it's the same size'. In an attempt to ascertain her concept of an angle she was asked 'what are you measuring with a protractor? Is it how long this arm is?' She replied 'Yes, I know, put it in the middle and see where this line is' indicating a complete misunderstanding of both the question and meaning of the scales on the half protractor. She had no idea of how to measure nor of the correspondence between the numbers on the protractor and the size of the angle.

She did not understand the notion of a full turn and when asked to perform one herself had to be prompted to turn beyond a half turn. She had no idea of the degree measure of a full or a half turn but used a half protractor to find that the right angle measured 90°.

She guessed 9(a) and 9(b) as 70° saying that they looked like the angle she had measured previously. When it was pointed out that 9(a) and 9(b) looked different from each other, she said 'It is same because it doesn't matter if it is different shape, it matters on these lines'. When asked if she meant the length of the lines she said 'Yes'. Her great confusion here is again hard to interpret as her usage of vocabulary is too unspecific.

She was unable to attempt item 10, attempted to mark points at the end of the parallel lines in item 11 and suggested 1000° for item 12.

She was given a full protractor, which she had not seen before and measured angle A on the sheet of angle drawings correctly as 30° because one of its arms was vertical. After 30 seconds attempting to measure B she placed the zero line approximately as the angle bisector which was not in a vertical direction, and read off 25° because one arm of the angle passed through the 25° mark. She was asked to measure E, an obtuse angle, and obtained 85° with the zero line approximately in the direction of the angle bisector and the hole about 1 cm away from the vertex.

Summary

Rifait is a weak pupil who attempts to rely greatly on memorising rote learned information. She finds difficulty in interpreting simple questions and lacks confidence when she is uncertain of what is required of her. Her responses are unspecific. She is confused about the criteria for comparing angles but feels that arm length and how far the arms are spread apart are important. She has not met reflex

angles nor does she possess an understanding of degree measure. She attempts to use the protractor as a tool to produce magic ready made answers and mistakenly feels that they are meaningful. She has no concept of measurement.

V A N D A N A

Vandana answered 1 and 2 correctly but selected (d) for item 3. She said that she had not previously met reflex angles. She initially mixed up the words acute and obtuse but later remembered them correctly. She named the right angle and confidently pointed out many examples in the room.

She correctly found 5(c) and 5(d) equal, and 5(e) and 5(f) equal, but stated that 5(g) was larger than 5(h) because the space between the top of the arms was greater. Referring to the earlier explanation of reflex angles in question 3 she found 5(i) greater than 5(j). She correctly found 5(k) = 5(l).

Initially she stated that a full turn was 180° and a half turn 90° but she saw the inconsistency when she also called the right angle 90° and quickly corrected her errors.

Her guesses for 9 were 40°, 80°, 170°, 180° and 340°, her only error being 180° instead of 270°. She initially said she had not met angles in a triangle and incorrectly guessed 50° for item 12 justifying this by saying that $100^{\circ}-50^{\circ} = 50^{\circ}$. She recognised the type of question but had forgotten exactly which angles to add and subtract. This indicated that she memorised rather than understood certain topics. In item 11 she pointed unspecifically to a vertex then chose two incorrect angles before finding a correct one.

She said she had not seen this type of diagram before.

She used a half protractor to measure A as 30° correctly and then D as 166° until the realisation that it was acute prompted her to suggest 26° which was near to the correct value of 29°. Her initial answer indicated that she was reading 154 as 166; i.e. reading the clockwise scale in an anti-clockwise direction.

She measured F as 134°, the largest sensible value directly measurable with the half protractor. When asked to approximate the size of F she suggested 170°. She was then issued with a full protractor and shown that it was rotatable. She measured A correctly as 30° and was then instructed how to read clockwise and anti-clockwise rotations and given interpolation practice. When measuring F she initially placed the zero line facing North and later placed it correctly reading off 311° instead of the correct 314°. When questioned about which type of protractor she found easier, she chose the full protractor.

Summary

Vandana is an able pupil but a little too keen on memorising facts and methods rather than on understanding. This may have been due to her schooling in India prior to the 4th year at primary school in England.

She was well able to estimate acute and obtuse angles with which she was familiar but not to extend this to reflex angles. She has met, but confused, angles in a triangle. She still makes mistakes in interpolating from scales and in reading in the correct direction indicating that, although she has acquired the skill to some degree, she does not have

the deeper understanding of the concept of angle measurement.

Surprisingly, in spite of her experience with angle work, she still felt that the distance between the ends of the arms of an angle could be an important factor in determining its size. This reinforces the opinion that she does not fully understand the concept of angle even though she can perform many of the tasks demanded in school correctly.

W E N D Y

Wendy answered 1 and 2 correctly but felt she had not previously been asked to order angles. She said 'we just did about acute and obtuse' indicating that she could remember categorising angles. She answered 3 correctly and, when asked what criterion she had used, answered 'how wide it is'. When asked to clarify this she pointed at the pink arc.

She had not met reflex angles before, but was able to point out an acute angle after some encouragement and to name 1(b) as a right angle. She initially selected 3(c) (reflex) as an obtuse angle but later chose a correct example.

She named the right angle in 4. In 5 she named (e) and (f) as right angles and hence equal. When asked how she had decided that 5(c) and 5(d) were equal she said she had looked 'down here' pointing towards the vertex. Closer questioning revealed that she had looked 'where you put the red loop' rather than at the sharpness of the point. She correctly found 5(g) and 5(h) equal and 5(i) larger than 5(j) because 'it's got a bigger loop'. She also correctly found 5(k) and

5(1) equal.

She answered 6, 7 and 8 correctly, saying that she had learned about degrees before. She hesitated for a long time in answering all parts of item 9, finally selecting 70° rather inaccurately for 45°, then 80° correctly, 120° for 156° and 70° for 270°. She was unable to see a method for improving upon this last answer when given time for thought and could not attempt the final reflex angle. She said that she had not guessed angle sizes before.

In item 11 she first marked the opposite angle of the parallelogram correctly and then marked another interior angle of the parallelogram incorrectly. She said she had not seen large diagrams containing many angles before.

Initially she was unable to answer item 10 stating that she had not learned about triangles but she correctly answered '30°' for item 12 explaining 'you add these two up and then find out how much to make 180°'. She described the calculation in terms of counting on rather than as a subtraction. After discussion she was able to answer item 10 correctly.

She had previously used a half protractor and measured 1(a) (50°) with it as 40° because she placed the 90° line on the vertical arm. She had not seen a full angle indicator before but, after having been shown that the top disc rotated, measured 1(a) as 40° too because the hole was not exactly over the vertex. With the half protractor she measured 1(d) (65°) as 20° because the 90° line was on the vertical arm and the mid point of the base of the protractor was a little above the vertex. She measured 1(d) correctly with the full

protractor.

With the half protractor she measured 2(c) (118°) as 170° because she placed the 90° line vertically. She also measured it incorrectly with the full protractor where she placed the 0° line vertical. She was asked why this position was correct and replied 'Don't know' but, when asked she said that she had previously made many measurements relating to compass directions. The correct method for measuring was explained to her and she was given a full protractor to look at to help her to estimate angles in item 9. She was now able to estimate both 160° and 270° correctly.

When asked twice whether angle was a measure of arm length, area between the arms or the amount of turn from one arm to the other, she selected the turn. She said she found the full protractor easier to use than the half protractor but could not say why. My apparent advocacy of it may have affected her response.

Summary

Wendy is a quiet and thoughtful girl with a wide vocabulary. She achieves a fairly high standard of work when not under pressure. She seemed faimilar with right angles and the degree measure of their multiples. She was able to compare angles correctly and may perceive them as rotations because she concentrates on the arc length marked at the vertex. It is not clear how she would respond to an arc of larger radius being drawn. In measuring she is both inaccurate in positioning the protractor over the vertex and unclear where to measure from. For a girl with a grasp of the angle concept her understanding of the meaning and purpose

of measuring angles is extraordinarily low.

P A U L

Paul answered all of items 1 to 9 correctly. He explained that he looked at the red loop and the sharpness in order to compare angles. He had learned about reflex angles before but called them 'abuse'. He recalled the word 'acute' but became confused when trying to remember 'obtuse'. He had also met degree measure to 360°.

He initially answered item 10 as 360° and when told this was incorrect, answered 180°. He recalled cutting off corners from a triangle and forming them into a straight line at primary school. He answered item 12 correctly. In item 11 he first selected an incorrect interior angle of the parallelogram and then two correct angles, the vertically opposite and the opposite corner of the parallelogram.

He had used both a half and full rotating protractor before. He measured various acute and obtuse angles correctly with both indicators. He said that he preferred the full protractor to the half because 'you don't turn this around miss, just put it on the point and then turn it round'. This explanation referred to the fact that the circular protractor was easily placed with its zero line on an arm of the angle and hole over the vertex, and looked almost the same whatever orientation it was in. Rotating the arrow then yielded the answer easily. The half protractor often had to be placed the 'wrong way round', in an asymmetrical position, in order to align its zero line with an arm.

To the question 'When you are measuring angles are you measuring length or area in this space or how much you turned round? he answered 'How much you turned round.' He recalled actually rotating himself during lessons on angle. He named clockwise and anticlockwise senses and realised that the measure of angle 5(g) was the same whether measured in a clockwise or anticlockwise sense.

Summary

Paul is a very bright boy, possessing all concepts tested. His error in item 11 suggests that he cannot always apply this knowledge in new contexts. He has summarised some of the difficulties in orientating the half protractor which are overcome by the use of a full protractor.

S H A U N

Shaun answered items 1 and 2 correctly. He suggested 3(b) because 'the circle's going on the outside'. When shown 3(c) was also reflex he selected 3(c) as the largest. Perhaps he had previously only considered 3(b) and 3(d), the angles with arms spread far apart, as possibly the largest and selected 3(b) because its arc was 'outside' while 3(d)'s arc was 'inside'.

When asked which was more important the circle or sharpness of the angle he selected the circle, although he had never met this type of angle before. He ably picked out examples of right angles in the room. He answered all of item 5 correctly explaining that the reason why 5(i) was larger than 5(j) was that the loop instructed you to measure

all the way round. When asked he showed how this could be done and measured 5(i) correctly with a full protractor. When asked if arm lengths mattered in measuring angles he said 'no'.

He answered 6 and 7 correctly but suggested 270° for item 8 giving the bearing of one arm rather than the size of the angle. When asked to measure with the full protractor he turned it the wrong way and still read off 270°, ignoring the arc. He finally realised that it was 90° but the influence of bearings or measuring from the vertical in school was so strong that it was carried over into inappropriate situations. He had previously used a rotatable full protractor.

He was reminded of the appearance of a 90° angle before embarking on item 9 which he answered correctly. He also measured 9(a) (45°) and 9(c) (160°) correctly with a half protractor. He said he had not guessed angle sizes before. He answered 360° for item 10 but corrected this to 180° when informed that it was incorrect. He answered items 11 and 12 correctly. He measured 3(c) (320°) as 315° using a full protractor but as 140° using a half protractor. The inconsistency had to be pointed out to him.

When asked which sort of protractor he preferred he chose the full one for the reasons that it could measure bigger angles and was easier to use, although he did not expand on the latter. To the question 'when we measure angles do we measure the length or the area or the amount turned?' he replied 'amount turned'. He said he had 'done turning' in school a long time ago.

Summary

Shaun is an able pupil with a clear idea of comparing and ordering acute and obtuse angles. He percieves angles as a rotation when measurement and comparison are required but is still unsure whether angle measure is free or from a fixed direction in the manner of a bearing. He is not certain that measuring and intuition should coincide or that degrees are an invariant unit of measure as indicated in his response to his two different results in measuring 3(c) with a full and a half protractor.

I B R A R

Ibrar answered items 1 and 2 correctly but stated that he had not done questions like this on ordering angles before. He selected 3(d) because 'its face is bigger'. His level of spoken English is poor and he may have meant to use a different word from 'face'. Closer probing yielded the comment regarding 3(a) 'this is too closed' and regarding 3(d) 'this is open' indicating that the spread of the arms was his criterion. It is not clear whether he saw this spread as the result of a rotation or as a static separation. He had explained correctly why the obtuse angle 3(d) was greater than the acute angle 3(a) and was then asked to consider the reflex angles 3(b) and 3(c). He selected 3(b) as larger because its arms were more open. When asked what the red loop meant he replied 'You must start here and end here' while pointing to the appropriate arms, almost implying a rotation.

He was given a full protractor to aid his explanation about 3(b) and, after a time spent in deciding how to position it, measured the angle (252^{o}) as 255^{o}. He then measured 3(c) (320^{o}) as 315^{o} because the hole was not placed exactly over the vertex. After this he was again asked which of 3(b) and 3(c) was the larger and he replied 3(b), rejecting his measurements in favour of his intuition. It was re-iterated that 3(b) was only 255^{o} while 3(c) was 315^{o} then he suddenly said '3(c) is bigger'. He had not met reflex angles before.

He could not give an answer for 4 even when the right angle was shown in other familiar positions. When asked if he could see examples of such an angle in the room, he made no response. It was then suggested that he looked at the piece of the paper on which the test was written and he pointed to 1(b), a drawing of a right angle. The test paper was turned over to its blank side in an attempt to make him concentrate on the corners of paper itself, but he looked through it and pointed to the drawing in item 4 which could be seen through the paper. A completely blank sheet of paper was then presented to him and when asked 'is there an angle the same on this paper?' he answered 'No'. He was shown the right angles on this piece of paper but it seemed that his experience of angles was restricted to pictures of intersecting rays and that he was unable to accommodate any other concept.

He selected 5(c) as bigger because 'one side is small and one side is bigger'. It is not clear why the small side

is referred to in this explanation. He selected 5(f) as bigger because 'is 2 sides the same, one side is down and one along' and when asked why this made 5(f) bigger than 5(e) described the latter as 'one side is long and one small'. He was totally confused regarding the aspect of each angle which he was required to compare but when reminded that he had thought previously that the openness of the arms was significant he was able to say that 5(e) and 5(f) were open the same amount.

He chose 5(g) as larger because its arms were longer, and 5(i) as larger but could not explain why. He selected 5(k) as larger because 'this is one arm is small and one bigger and this is two same'. When asked whether it was the big or small arm of 5(k) which was important he answered that it was the big arm. As in 5(c) it is still unclear why the small arm is referred to.

He answered 6 and 7 correctly by looking across at the full protractor placed in front of me on the desk. This intelligent approach by such a weak pupil was surprising; particularly since he was not initially able to mentally overlay the right angles for comparison in item 5(e) and 5(f). He was unable to answer 8 in which the right angle was drawn in the second quadrant. By looking across at the full protractor he was able to estimate correctly all but the fourth part of item 9, even though some of these did not have a vertical arm. He could not attempt items 10 or 12 and marked two interior angles in item 11, one correctly and one incorrectly.

When asked to measure 2(c) (117°) with a half protractor he was unable to position it correctly and read off 180°, but with a full protractor he read off 120° after almost saying 240°. When asked to measure 1(a) (48°) with a half protractor he answered both 40° and 140° having placed the 90° line on the vertical arm. To check this he was given a full protractor which he slowly aligned with the hole over the vertex and used to obtain the answer 50°. 2(a) (26°) was measured as 60° with the half protractor because the 90° line was placed on one of the arms but it was measured as 30° with the full protractor. He stated that he preferred the full protractor.

Summary

Ibrar is a very slow pupil with additional language problems. He does not express his reasons clearly and perhaps does not even possess clear criteria for each decision he makes. From his school work he has retained the idea of sharpness, as evidenced in his use of 'open'. This notion is insufficiently clear in his mind to prevent him from feeling that arm length, and perhaps the length of the one longest arm, affects angle size. He may even feel that he is expected to compare different attributes of an angle in items 1 to 3 where all arms are equal and item 5 where they are not. He may not see angle size as one consistent attribute of all types of angle.

He had no concept of the relationship of angle measure to intuitive notions of size. His concept of an angle as a pair of rays has compartmentalised it to within certain classroom mathematical experiences and he has been unable to

relate these to other forms of representation or to observe concrete examples of angles.

His usage of the full protractor which happened to be visible for the estimation of angle size was very sensible. It has suggested this approach as a useful stepping stone towards angle estimation in terms of mental images. It may be that Ibrar did not see the process as estimation but as exact measurement because he was not aware of the importance of accuracy in the measurements which he performed under observation.

The full protractor also enabled him to measure angles which he found impossible or measured incorrectly with the half protractor.

4.6.3 SUMMARY AND INTERPRETATION OF INTERVIEWS

The primary interviews brought to light valuable insights into both pupils' primary school experiences of angle and their misconceptions. Helpful comments and suggestions also emerged. A summary and interpretation of these is given.

It was apparent that certain notions and activities had not been met in primary school while others had been stressed.

Reflex angles, drawn as static ray pairs, were unfamiliar to most pupils. The words 'acute' and 'obtuse' had been met but were frequently not recalled or recalled incorrectly, even by brighter pupils who could deal ably with the concepts. The decision, discussed in section 3.2, to reduce technical terminology by the omission of 'acute'

and 'obtuse' from the written test was seen to have been a wise one.

The greater number of pupils comparing the sharpness of angles rather than the arcs at their vertices indicated that a static approach to angle was prevalent in the primary schools, although it is clearly more natural to relate a drawing of an angle to the static than to the dynamic approach.

Angle size had been measured in degrees but not estimated. Few pupils had seen rotatable full protractors yet all had used half protractors to measure angles. The most startling revelation from the interview session was that, in spite of this, the pupils preferred using the full protractor and were far more successful in measuring with it.

Serious misconceptions relating to the attributes of an angle considered to affect its size were revealed.

Some pupils utilised notions such as sharpness and arm length seemingly randomly for the comparison of different pairs of angles, indicating a severe misunderstanding of the need for a consistent set of criteria for determining the relative size of angles. Incorrect attributes used in comparing angle sizes were the length of one or both arms and the space between the ends of the arms. Two pupils, when comparing equal angles one of which had two equal medium length arms and the other of which had one arm longer and one arm shorter than these, made specific reference to the shorter arm in addition to the longer arm in selecting this angle as the larger. It is not clear why any reference was made to the shorter arm. The emphasis which one pupil placed on the arc

length rather than on the rotation which it represented prompted the idea of the inclusion in the secondary test of an item in which equal angles were marked with arcs of different radii.

Another serious problem highlighted was that most pupils made no connection between angle measurement and the concept of angle size. The APU [18] and Holloway [10], as discussed in section 3.2, have also noted such misunderstandings. One girl measured two angles as equal but still used her intuition in stating that they were unequal. A boy measured one angle as larger than another but initially rejected this in favour of his visual judgment in finding it smaller. An able boy was not surprised by his different answers obtained from measuring the same angle with different types of protractor. A common error was the selection of the wrong scale in reading off the angle size and some pupils even offered both values simultaneously as correct. Errors of these types indicated that pupils of all abilities viewed angle measurement as a skill separate from notions of angle size and that degrees were not considered as an invariant unit.

There were inaccuracies in measurement and erroneous methods used. The frequent inaccuracies of up to 5°, partly due to previous experiences of measuring angles the sizes of which were multiples of 5°, were accepted without requests for remeasuring as it was considered far more important to identify the more serious conceptual errors.

When using a half protractor the 90° line was sometimes placed on one arm or vertically or along the angle bisector.

When using a full protractor the 0° line was sometimes placed vertically, as if a bearing was being measured, and at other times placed along the angle bisector. Some of these positions were due to previous experiences of measuring angles with one horizontal arm using a half protractor in which its correct positioning was with the 90° line vertical. This orientation was one which pupils were likely to view as 'the right way up' because it was the most usual orientation required in early measuring exercises and because illustrations of the half protractor usually show it this way up. Pupils often did not move the half protractor from this symmetrical, and therefore visually pleasing, orientation. The comments of one pupil drew attention to this point and the fact that the apparently symmetrical appearance of the full protractor, its zero line and numbering being very pale, rendered it easier to position over any angle arm than the half protractor which had to be turned into an undesirable asymmetrical position. In spite of this angles were sometimes measured with a full protractor starting from a vertical direction. This could be due to a confusion between angle size and arm position as in a bearing or to the desire to position the protractor symmetrically.

Another type of measurement error was that of incorrect interpolation on the scale. Some pupils were unaware of the purpose of the marks between the numbers on the scale, indicating that they did not understand measurement of length using a ruler. Others did not notice that the scales were numbered in opposite senses and interpolated in the wrong direction answering, for example, 166° instead of 154° for

an angle 6^{o} away from 160^{o} but smaller than it.

It was found that more pupils could point out examples of right angles in the room than could name them. This indicated the need for an item testing the ability to recognise examples of right angles in the written test, a finding borne out by the primary test results. One pupil felt that right angles were always on the right side of the page. Another exhibited difficulty in identifying right angles other than those drawn as ray pairs i.e. angles composed solely of two rays emanating from a vertex. Even when corners of a page were pointed out to him he could not see them as right angles. This suggests that there has been concentration on only one form of static angle, the ray pair, rather than on as many forms as possible including corners of shapes and objects.

When calculating the third angle of a triangle one pupil exhibited the method of adding on rather than of adding the two given angles and then subtracting. This justified the use of easy angle sizes in the triangle which had been provided in order to enable pupils using this method to answer correctly and thus to exhibit their understanding of the concept being tested. In K.M. Hart (ed) [19, page 29] attention is drawn to the difficulty pupils experience in interpreting such questions as requiring the operation of subtraction. None of the three pupils who answered this item correctly had stated the angle sum of a triangle correctly at their first attempt. This indicates the unreliability of abstract questions in revealing whether a fact is known.

One pupil's use of glancing at a full protractor placed elsewhere on the desk in order to help him to estimate angle size suggested the use of such a method as an intermediate step towards estimation by comparison with a mental image.

Most pupils felt that angle was a measure of turning when offered this as an alternative to a measure of length or of area.

During the course of the interviews it became clear that questions relating to the equality of clockwise and anti-clockwise measures of an angle could not be put to most pupils either because of their lack of understanding of the relationship between measurement and angle size or because of their poor level of language comprehension and expression. One bright boy was, however, immediately able to state that the measures were equal.

In addition to indicating the extent of the anticipated misconceptions, the primary interviews brought to light some unexpected ones. They highlighted a range of errors which could be expected, with both types of protractor, when pupils attempted to measure angles of various sizes and in various orientations. The misconception that angle size depended upon arc length, irrespective of its radius, was also brought to light. These particular findings prompted the construction of new items for inclusion in the secondary test in order to ascertain the extent of such errors and misconceptions. Other findings, considered in the light of the primary test results, assisted in the selection and emendation of certain test and interview items for use in the secondary tests.

CHAPTER 5

ANALYSIS OF THE MAIN PRIMARY TEXTS USED

The reasons for undertaking the text analysis are given. The main texts and the manner in which they are incorporated into the work on angle are described. The general approach to angle taken by each text is outlined and then detailed description of the presentation of angle concepts and introduction to angular measurement is given. Arising from the critical discussion of content it is seen that improvements in both the primary texts and the classroom presentation are required. Lists of recommendations for both of these are made. In addition the insights into the test and interview responses afforded by the text analysis are discussed.

5.1 REASONS FOR THE ANALYSIS

Each pupil's test and interview responses were not only dependent upon their understanding of angle but also upon the presentation and approach to angle with which they were most familiar. Clearly the extent of the work on angle which they had undertaken also affected their responses.

Information regarding the pupils' primary school experiences of angle would therefore assist in the interpretation of the test and interview responses.

Discussions with the primary teachers in charge of mathematics had revealed that the approaches used and details of presentation varied from teacher to teacher. It would not

be possible to ascertain the exact nature of each teacher's presentation of angle or to relate the results of each individual pupil to the methods of their primary teachers.

The discussions with primary teachers also revealed that each school used a number of texts, sometimes referring pupils of different abilities to different ones. Most text series consist of four books, one intended for each primary school year, but some provide an additional book for older pupils. The teachers were able to outline policies for the issuing of these texts and to state the highest book number in each series used by their fourth year pupils.

It would be possible to analyse the angle content of the main texts used. It would not be practicable to relate each individual pupil's responses to the texts which he or she had used but the overall findings of the analysis would provide a useful frame of reference in which to interpret all test and interview responses. They would also enable recommendations for improvements in the presentation of angle in primary school to be made.

Accordingly it was decided that the main primary texts used would be analysed for their approach to the angle concept and angle measure from its initial introduction to the work intended for fourth year primary school pupils.

5.2 THE MAIN TEXTS AND THEIR USE

The four main feeder primary schools based their work on a variety of texts and class lessons. They had all taught the measuring of angles using half protractors as class lessons. Only one school worked almost exclusively from a

single scheme, the SMP 7-13 workcards, but even they enriched this with various texts and class lessons.

The five main schemes used were: Primary Mathematics (PM), Stages 1, 2, 3, 4 and 5: SMP 7-13 workcards (SMP), units 1, 2, 3 and 4; Maths Adventure (MA), Books 1, 2, 3 and 4; Making Sure of Maths (MSM), Books 1, 2, 3, 4 and 5; Alpha Mathematics (AM), Books 1, 2, 3 and 4.

Some teachers issued a new text each year and thus parts of each book were often omitted. Other teachers set work appropriate to each child's ability and consequently many pupils did not reach beyond book 3 in a series. Book 5 was rarely used and often only half of book 4 was completed. It was therefore not possible to tell which parts of the work on angles had actually been met. It was common for different topics to be taught from different texts and for extension work from texts, such as AM, to be issued only to brighter pupils when they had completed the work on the same topic in another text. A combination of class taught topics and individual or group work was most common but some teachers favoured only one type of lesson format.

5.3 OUTLINE OF THE APPROACH TO ANGLE IN EACH TEXT

Angle is given different degrees of importance by each text. The quantity of work on angle varies from MA, which includes very little, and AM, which offers very few examples in each of its angle related sections, to SMP which has 67 workcards, albeit including revision examples at the beginning of each section in units 2, 3 and 4.

The presentation of the work varies. PM and SMP both have long specific angles sections under the general heading of shape. The other three texts have frequent one or two page sections on angle related topics.

The approach to angle also differs from text to text.

MA always uses rotations, an approach which is justified because of the primary nature of the notion of rotation met in everyday life movements. The teachers' notes refer to the many dynamic contexts in which pupils now meet angle, such as the constantly changing bearings in space flight. Angle is introduced for its practical uses. A large amount of work on bearings is given, including the plotting of fairly complex courses.

SMP also starts with a dynamic approach and later offers a wide variety of both static and dynamic examples. There is frequent revision of earlier units of work and the presentation is clear. The work extends to the accurate drawings of triangles and quadrilaterals and calculation of unmarked angles. Very little work on bearings is introduced. Pupils' interest is developed by examples such as those in which astronauts using computers draw triangles to locate positions in space.

PM uses a static approach to introduce angles. Prior to this shapes are recognised and named intuitively. Angles are then defined as corners of these shapes. Rotations are later introduced in compass and clockface examples although the static approach is emphasised throughout. The presentation is fairly clear. Once the basic concepts have been established angle is used in the classification of polygons.

The angle experiences given therefore start and finish with polygons.

AM uses a traditional static approach to angle following the definitions of oblique, parallel and perpendicular lines. A high level of language is used throughout. Abstract symbolism and the labelling of angles with three letters is introduced. Rotations are referred to briefly in compass work and indicated by directional arrows on the arcs drawn at angle vertices. Many topics are introduced but insufficient exercises are set on each one. Harder notions such as alternate angles, the exterior angle of a triangle and identification of types of triangle are also introduced. The text can be used for extension material for able pupils although little explanation is given.

MSM uses a similar approach to that of AM throughout. It is almost merely a simpler version of AM in that harder topics are not introduced and more explanation and examples are provided.

5.4 THE PRESENTATION OF ANGLE CONCEPTS

The presentation of various angle related concepts in each text is described in a developmental sequence.

It must be remenbered that few pupils reach the final book in a series. Where the position of a topic within the primary course is deemed important the number of the book is written in brackets after the series title.

Introduction of the right angle

The earliest common angle topic in all texts is the right angle. It appears in book 1 in all except PM where it appears in book 2. The words 'square corner' appear as a synonym for a right angle in all except MA. The right angle is defined in various ways.

PM introduce it as a corner of a square, MSM and AM state that where a vertical and horizontal line meet they make a square corner which is also called a right angle, SMP states that both a quarter turn and a square corner are a right angle and MA introduces a right angle as a name for a quarter turn.

Primary notions

Prior to these definitions the texts have introduced different elementary notions. PM starts with intuitive shape recognition. MSM starts with vertical, horizontal and oblique lines. AM starts with straight, curved, vertical, horizontal, oblique, perpendicular and parallel lines as well as shape recognition. SMP starts with rotating oneself through multiples of full turns, half and quarter turns, clockwise and anti-clockwise and turns on street maps. MA starts with a very small amount of work on full, half and quarter turns.

Development of right angle concept

Those texts which introduced the right angle as only a static concept PM, MSM and AM, defined it as a rotation in

the following book. All texts used a 'right angle tester' to overlay on diagrams or to fit into corners in the room. It was always made by folding paper in half and then half again in such a way that the first fold fell onto itself. Some texts advocated the use of a circular piece of paper as its edges could easily be aligned when folding. As this can create a misconception that the right angles only occur in quarters of circles some texts folded an irregular piece of paper. The findings of the APU (see page 24) show that this too is misinterpreted. Texts folding scrap paper were SMP, MA and MSM; those using a circle were PM, AM and MSM (after their scrap paper example).

The concept of a right angle was reinforced often with the self made tester, but too frequently on right angles oriented square to the page. The tester was placed in positions or over corners for comparison or checking. PM used a tester to distinguish between quadrilaterals; made right angles on nail board; tested for them among ray pairs, in triangles and in mazes, in which they also counted them. It also tested ray pair angles as larger or smaller than right angles. MSM explained that page corners were right angles, tested and counted the number of right angles in figures and tested whether ray pair angles were larger or smaller than right angles. SMP tested and counted right angles in polygons and practised clockwise and anti-clockwise rotations through 1 to 8 right angles. MA tested the room and book for right angles and sought objects which rotate through a right angle. AM tested polygons for right angles, drew right angled triangles and tested ray pair angles as

larger or smaller than right angles.

Sense of rotation

The words 'clockwise' and 'anti-clockwise' are sometimes assumed to be known and introduced rather late. They are first used relating to clocks and compasses by PM and MSM in book 3 and by AM in book 2. SMP introduce them in unit 1 for half and quarter turns and later with compasses. AM does not mention them.

Naming full and half turns

The phraseology 'full turn' is used in SMP and MA book 1 and in PM book 3 which also uses 'complete revolution'. MSM book 3 uses 'complete revolution'.

The phraseology 'straight angle' is used by PM and SMP who introduce it as follows: 'An angle of 180°, or two right angles, is called a straight angle'.

Introduction of acute and obtuse angles

PM and MSM define acute and obtuse angles by intuitive comparison with right angles whereas SMP and AM first introduce degrees, measure the angles and then define acute and obtuse angles in relation to 90°.

Comparison of size by inspection

PM is the only text offering practice at intuitive comparison and ordering of angles although MSM does include comparison of angles with a right angle for categorising them as acute or obtuse.

Reflex angles

These are only named in one text, AM (3), where they are used in an exercise on calculating angles at a vertex. In other texts they appear as rotations of between 180° and 360° in mixed exercises with smaller rotations and in bearings. Thus experiential foundations are laid but no categorisation is attempted.

They appear as ray pairs in AM and as interior angles of concave polygons in PM (5), which is rarely reached by any pupil.

5.5 THE INTRODUCTION TO ANGULAR MEASUREMENT

The content of each text relating to the stages of the introduction of measurement is described.

It must be remembered that few pupils reach the final book in a series. Where the position of a topic within the primary course is deemed important the number of the book is written in brackets after the series title.

Right angle and half right angle as units

Some texts measure rotations in right angles using abstract diagrams (SMP), clock (PM) or compass directions (MA). PM also uses right angles to measure interior angles of convex polygons.

The half right angle is used as a unit of measure in clock rotations by PM and in both abstract rotations and polygons by SMP.

Introduction of the degree

Three texts make a brief reference to history and attribute the concept of degrees to the Babylonians. Three texts start with 360° while one subdivides the previously introduced right angle unit and another stresses the small size of 1°. They all use the $^{\circ}$ symbol, MA doing so without explanation, however it does stress the convenience of 360 due to its many factors. The wording used in each text is of interest and is reproduced below.

PM (4) 'Angles are measured in units called degrees. A degree is a very small rotation. This angle measures one degree. We can write this as 1°.'

Figure 4

MSM (3) 'We know the circumference of a clockface is marked off in 60 small divisions, called minutes. But let us imagine it is divided into 360 small divisions which we call degrees. For short we write 360 degrees as 360°.'

SMP (3) 'A long time ago we Babylonians wanted to measure angles very accurately. We divided a right angle into 90 parts. Each part we called a DEGREE. A short way of writing 90 degrees is 90°.'

MA (3) 'The ancient Babylonians had 360 directions in one full turn. Nowadays we call them degrees.'

AM (2) 'Babylorians used 360 degrees (360°). The same unit of measurement is still used today for measuring turns or rotations.'

Initial marking of angles to be measured

MSM, AM and MA initially mark angles using arrowed loops which indicate a dynamic aspect of measurement. PM uses a shaded loop which boldly distinguishes the acute angle from the reflex angle and enables the angle to be seen clearly through the protractor. This shading highlights the sharpness of the point while the arc of the sector also represents the rotation. SMP starts with a green loop and changes to a shaded loop and completely shaded polygons in later work. MA uses no marking for the interior angles of polygons.

First uses of degree measure

Degrees are first used by all texts in rotations on a marked wheel, compass or clockface. PM (4) first introduces rotations of 0°, 90°, 180°, 270°, 360° and then in steps of 45° on a compass and 30° on a clockface. MSM (3) first introduces, as fractions of 360°, 90°, 180° and 270° then 45° by folding a square. It uses rotations in steps of 45° on a compass and 30° on a clockface. SMP (3) introduces multiples of 90° and 45° turns on a wheel. MA (3) first uses degrees to measure turns marked in multiples of 15° and then in multiples of 30° on a clockface. AM (2) shows a full circle marked every 10° and makes turns in multiples of 90° and then of 45° on a compass.

Non-standard protractors

All texts except SMP introduce the notion of a protractor using an instrument with a small number of markings. PM show

a half protractor with spokes every 10° and markings only at 0°, 90° and 180° to encourage counting and interpolating for angles that are odd multiples of 5°. MSM provide two half protractors to be traced. They have marked spokes every 30° and are numbered in opposite senses. Initially pupils are told which one to use for each angle to be measured but later they must first select the appropriate one. They draw and measure angles in multiples of 15°.

MA provides incompletely marked full and half protractors in its workbook, to be numbered in steps of 10°. Angles in multiples of 5° between the spokes of wheels and in very large triangles and quadrilaterals are measured. AM explain how to mark a semi-circle in steps of 45° and a circle in steps of 30° by folding. These are used to measure and to draw angles which are multiples and sums or both of 30° and 45°.

Standard protractors

PM (5), MSM (5), SMP (3) and AM (3) introduce half protractors and PM (5), SMP (3) and AM (4) introduce full protractors. SMP leaves the explanation of how to use them to the teacher. PM and AM encourage prior estimation of angle size. MSM and AM encourage prior observation of whether the angle is acute or obtuse to facilitate choice of the correct scale.

Orientation of the first angles measured with standard protractors

All texts first use the half protractor to measure ray pair angles with one arm horizontal and give more examples

with angles in this orientation than of obliquely oriented angles. SMP first uses the full protractor to measure such angles while PM first uses it to measure angles with one arm vertical and AM for bearings.

Angles measured with standard protractors

PM (5) measures to the nearest degree angles at the centre of a circle, then ray pairs in all orientations then interior angles of polygons including reflex angles. MSM (5) measures to the nearest degree ray pair angles in all orientations, angles in triangles and at vertices. SMP measures first to the nearest 10° (3) and later to the nearest degree (4) ray pair angles in all orientations, then angles of triangles and quadrilaterals. AM (4) measure to the nearest degree ray pair angles in all orientations and then angles at vertices.

Drawing angles

SMP (3 and 4), AM (4) and MSM (5) introduce drawing angles. SMP and MSM extend this to the drawing of triangles.

Angle sum of a triangle

All texts introduce the angle sum of a triangle but at different stages: AM in book 2, MA and SMP in book 3 and MSM and PM in book 5. MSM and SMP illustrate this by tearing off the corners of the triangle and forming a straight angle with them. They verify it by adding the measured angles and they also calculate unmarked angles of a triangle.

Bearings

MA (4) develops the notion of bearings from describing positions with a full protractor marked in steps of 10^{o} to measuring and plotting complicated courses. Other texts treat bearings lightly and usually late in their series. PM (5) uses a circle marked in steps of 10^{o} with North in various orientations on the page. SMP (4) uses only 8 compass points and one extension question on bearings in the playground. AM (3) uses a circle marked in steps of 10^{o} which is developed in book 4 with the use of a full protractor to measurements accurate to the nearest degree of bearings of children from various places on a map.

5.6 CRITICAL DISCUSSION OF ANGLE CONTENT

There is a diversity of approaches to angle in the main texts. Schools utilising more than one text should take advantage of this to offer the pupils a broader understanding of angle. Even within each text, however, static and dynamic approaches to angle are often not brought together to assist in the development of the concept of angle. The separately presented experiences with rotations, which are not recorded diagrammatically, and with drawings of static ray pairs must be associated. There is clearly a need to develop both static and dynamic approaches as equally important and to indicate to pupils that they both relate to the same concept, angle.

There are also important differences and inadequacies in the presentation of some of the individual angle topics.

The central mathematical and historical importance of the right angle is reflected in its early and thorough inclusion in all texts. The common use of angle testers helps to establish the static concept of a right angle but drawings frequently show them oriented square to the page. Use of the tester on angles other than right angles reinforces both the concept of a right angle and the concept of angle size, in that comparison with the tester can highlight the important criteria and generate discussion on the irrelevance of area and arm length. There should be more examples of this use of testers. Right angles (and angles in general) are also too frequently illustrated by ray pairs or two adjacent sides of a polygon, which can render any different representation of the angle, for example the corner of a room, hard to recognise.

The use of the synonym 'square corner' for 'right angle' can cause more confusion than benefit in confining the concept both to a certain quadrilateral and to a static interpretation. David Fielker (see page 23) or [17, p.108] feels that the introduction of a right angle in this way can result in it not being percieved as a particular case of a more general angle.

Acute and obtuse angles are identified AFTER being measured in two texts. Given the difficulty which pupils experience in relating measurements to their intuition, as discussed on page 25 , this may result in a lack of awareness of the bluntness of an obtuse angle and sharpness of an acute angle. Comparison of acute and obtuse angles should come from perception rather than measurement.

Reflex angles drawn as ray pairs have not been met by the majority of children, but they have performed rotations of between 180° and 360°.

A very small amount of work was devoted to practising clockwise and anti-clockwise rotations which are required, and often badly performed, in secondary school.

Each text did not define an angle because it was introduced at a very early stage. However the varying definition of degrees did highlight some subtle differences in each text's approach to angle. They included examples of a degree as a measure of rotation and as an arc length. As stated in section 2.2 explanations of the historical development of angular measure do not make it clear whether the degree was first seen as a unit of rotation or of arc length.

The PM definition, which illustrates the size of 1°, helps pupils to realise how small such an angle is. In visualising an angle of, say, 30° it is easier for pupils to imagine a number of these than to imagine a fraction of 360° because both large numbers and fractions are confusing for them. This definition does not, however, draw attention to the inbuilt measure of angle, a full turn, and its subdivision into degrees. The SMP definition of a degree in terms of a subdivision of a right angle also postpones the statement of the number of degrees in a full turn. The MSM definition seems to be introducing a personal rather than a universal unit due to its use of the subjunctive.

An ideal definition of degree would include reference to the Babylonians and state that there were 360° in a full turn. An accompanying illustration of 1° and some other angles would

assist in providing a feeling for the magnitude of these rotations.

The use of the historically prior and conceptually easier units of measure, the right angle and half right angle, is widespread. They serve as a helpful introduction to the concept of angle measurement but it should be stressed that they are not standard units when they are superseded by degrees so that pupils do not attempt to use these units for too long.

There is a lack of intuitive comparison of angles and insufficient work on estimation of angle size prior to measurement, even in the texts which attempt to encourage it. This can result in the skill of measuring being divorced from intuition. The use of sparsely numbered non-standard protractors which encourage counting is an excellent way to relate measuring to intuitive size.

The introduction of standard protractors concentrates on the use of the half protractor which is unable to convey the idea of a rotation when in use. It appears late in most texts and yet discussions with staff revealed that it has been met by most primary pupils. This suggests that teachers introduce it too soon in class lessons. Perhaps they mistakenly perceive angle measurement with a half protractor as an easily learned skill. The additional fact that many children own a half protractor as part of their geometry set may encourage its early inclusion in the syllabus.

All texts encourage use of a non-standard protractor or measurements in steps of 10^{o} on a standard protractor prior to measurement to the nearest degree. A protractor marked in

steps of 10^{o} or 30^{o} can assist in recognising the sense of a scale and in developing the skill of interpolation. It can be worked with for a long time until the measuring skills are developed. Only then should measurement using a standard protractor be attempted. Understanding of the concept of measurement and of the techniques required is far more important than attempting to measure to a greater degree of accuracy.

The majority of angles to be measured are ray pairs with one arm horizontal and consequently the notion that the size of an angle is a measure of the displacement of one arm from a fixed direction may easily be formed. At an early stage angles in different orientations and with different arm lengths must be measured so that an understanding of angular measure is developed by varying all the irrelevant attributes. The use of a rotatable full protractor would convey the idea of rotation, the essential attribute of angle. A large and varied experience of measuring angles is essential. The measuring of angles in triangles however is sometimes introduced too early. Unless triangles are extremely large the third side causes confusion when a pupil attempts to isolate an angle for measurement. MA attempts to overcome this by using extremely large triangles but lack of space forces these to be drawn overlapping each other, a format which may itself be confusing.

Different conventions are used for marking angles. Arcs marked with arrows most clearly indicate the motion involved. This type of introduction is not followed by sufficient examples to show that the measure is independent

of the sense before the use of an undirected arc is adopted. Because the radii of the arcs in any one exercise are equal, the size of an angle can easily be thought to be related to the area inside the loop, especially when the loop is shaded. This could be overcome by the use of arcs of different radius, initially on angles of the same size. Discussion of angles turned through by different points along a clock hand could be used to reinforce the idea and to relate rotation to its static representation.

5.7 LIST OF RECOMMENDATIONS FOR IMPROVEMENTS IN PRIMARY TEXTS

In the discussion of the angle content of the primary texts various suggested improvements which could aid understanding were described. These recommendations are listed here for quick reference.

1. Do not only define a right angle as a square corner.
2. Do not always orient early drawings of right angles square to the page.
3. Use the right angle tester to identify acute and obtuse angles.
4. Give more practice with clockwise and anticlockwise rotations.
5. Define and identify acute and obtuse angles by perception and not measurement.
6. Include in the degree definition a historical reference to the Babylonians, that there are 360° in one full

turn and a diagram to indicate the magnitude of 1°.

7. Give more examples of intuitive comparison of angle sizes during very early work on angle.
8. Always encourage estimation of angle size prior to measurement.
9. Encourage the use of full rotatable protractors by illustrating and using them.
10. Vary orientation, arc radius and arm length of angles to be measured right from the start.
11. Initially mark arcs with arrows and only introduce unmarked arcs when the equality of the clockwise and anticlockwise measure of an angle is understood.

Overall it is recommended that static and dynamic approaches are interrelated.

5.8 INSIGHTS INTO TEST AND INTERVIEW RESPONSES AFFORDED BY THE TEXT ANALYSIS

The analysis of the primary texts afforded interpretation of certain test and interview responses.

It was revealed that reflex angles, represented as ray pairs, were only met by very few pupils. The notion of reflex angles, introduced as rotations, but rarely recorded individually in diagrams, (see page 80) was however met by most pupils but a connection between this and its static representation in the test was not made. The inability of many pupils to extend their angle concept to reflex angles, exhibited by their responses to items 3 and 5 in the test,

implied that they possessed a static concept of ray pair angles.

Either the static approach to angle is emphasised by most primary teachers or the large amount of work on rotations in the primary texts, if actually worked through, is not related to the work with ray pairs in forming the pupils' concept of angle.

The low facility in items 6 and 7, the degree measure of a full and a half turn was all the more surprising when it was seen that the initial use of degree measure in all texts was in rotations through the first four multiples of 90°. The majority of texts used degree measure in this way in book 3 and so it should have been reached by most pupils. That they have met degree measure is evident from the fact that they have been taught to measure angles using half protractors.

Rotations in degrees were initially carried out on diagrams similar to eight spoked wheels in which no individual angles were marked and for which no diagrammatic results were drawn. In contrast the statement that there were 90° in a right angle was usually accompanied by a static ray pair diagram of a right angle. Difficulties in relating the degree measure of a right angle and of multiples of quarter turns may have arisen from the lack of emphasis that a right angle is a quarter turn at the time degrees were introduced. Insufficient emphasis has been placed on rotations of 90°, 180°, 270° and 360° and on the calculation of multiples of 90 in primary school in spite of the provision of appropriate examples in the texts.

The small amount of work on bearings appears late in each series and only one text draws them as ray pairs which have to be measured with a circular protractor. The remainder present a drawing of a circle subdivided into sectors and ask for the bearing of certain marked points to be read off from the diagram without the use of a protractor. Consequently very few pupils have experienced bearings drawn as ray pairs or measured them with protractors. It is therefore unlikely that pupils who always attempted to measure angles starting from the vertical did so because they were experiencing interference with the notion of bearing. It is far more likely that the orientation of most of the early angles measured and the desire for symmetry in positioning the protractor were causes of this type of error.

It seems sensible to omit work on the measurement of bearings until all the more basic concepts are understood.

All three areas of difficulty discussed above show that pupils experience problems in relating the various diagrammatic representations of and approaches to angle.

The text analysis revealed the widespread use of the right angle and half right angle as units of measure during the introductory stages of angle measurement. This explained responses in the range 1 to 5 given by three pupils to items involving degree measure. While they are undoubtedly a helpful stepping stone, the use of these non-standard units should be seen to be inappropriate when the question asks for answers in degrees. When degrees are introduced it should be stressed that they are, in contrast, standard units.

In view of the fact that most pupils had been taught to measure angles using a half protractor it was surprising that the performance in this interview item was extremely low. The text analysis has shown that most books use a good gradual introduction to angular measurement. The only reason for the low interview performance can be that pupils have not worked through these introductory stages sufficiently slowly. This evidence points strongly to the fact that primary teachers do introduce measurement with a half protractor before teaching all the prerequisite skills and concepts.

5.9 LIST OF RECOMMENDATIONS FOR PRIMARY TEACHERS

The interpretation of certain test and interview results in the light of the text analysis revealed areas where the teacher could effect improvements in understanding.

The recommendations for improvements in primary texts should be followed in teaching. In addition teachers should:

(1) Constantly relate the ray pair and rotation representations of angle,

(2) offer and interpret a variety of diagrammatic representations of angle,

(3) emphasise rotations of 90°, 180°, 270° and 360° and practise calculating multiples of 90,

(4) remind pupils that the right angle and half-right angle are non-standard units and that angle measures should be given in degrees as soon as they are able,

(5) adopt a gradual approach to angular measurement as outlined in the texts, and

(6) omit work on bearings and extend the use of the full protractor to the measuring of all angles.

CHAPTER 6

THE SECONDARY TESTS AND INTERVIEWS

The purpose of the tests and interviews is outlined in the first section. The decision to give two written tests and the content area of each test is described in the second section. Details of the administration of the tests and interviews and of the pupils used as subjects are given in the third section. The administration of the tests to different classes enabled certain hypotheses to be tested. These are described in the fourth section where an outline of the statistical tests used is also given.

In the fifth section each item in the Angles test and the responses to it are analysed, shortcomings of the test are pointed out and the results of classes are compared. In the sixth section each item in the Measuring Angles test and the responses to it are analysed, types of error are discussed and the results of classes are compared. In the seventh section the interviews are analysed and summarised.

A list of problems experienced by pupils is given. The misconceptions observed are summarised and recommendations are made for overcoming them.

6.1 PURPOSE

The secondary test was intended to exhibit quantitatively the level of understanding and wide range of misconceptions regarding angle with which first year pupils were entering my school. The interviews were intended to probe these more deeply. The findings would enable suggestions to be made for remediating the misconceptions and for items to be included in a new angles unit to be taught to all first year pupils. This unit was to be constructed to help the pupils to overcome their misconceptions and to develop a firm understanding of the basic angle concepts and measurement

skills prior to the introduction of more advanced work.

6.2 SELECTION OF ITEM AREAS

The primary interviews had revealed the severe difficulties which some pupils experienced in attempting to measure angles in spite of having been taught to use a half protractor. It seemed also that the same pupils, when handed a full rotatable protractor (see Figure 2, page 48) which they had never seen before, were able to measure more angles correctly with it. This factor could have been influenced by the inference that some pupils made, that I was advocating the use of the full protractor, causing them to try much harder with it. It was considered most important to ascertain whether proficiency in measuring with a full protractor was significantly better than that with a half protractor. A large scale written test on angle measurement was therefore required. It was practicable to administer it in my school where sufficient equipment was available. Consequently it was decided that two written tests should be constructed; one similar to the primary test and based on angle concepts, the other on angle measurement.

The first secondary test was entitled 'Angles' and the second 'Measuring Angles'. They appear in their original format on page 17 of the Appendix.

The items in the Angles test were mainly adaptations of those of the primary test with the addition of items from the primary interviews on the recognition of a right angle and

the definition of an angle. The harder primary test items were not used as bases for Angles test items. The intention of this test was to reveal the level of understanding and the misconceptions relating to angle concepts.

The Measuring Angles test asked if half protractors and full protractors had been met in primary school and provided six varied angles for measurement. It was intended that this test would reveal the level of proficiency in angle measurement with both full and half protractors.

The source of items in each test is shown in Table 2 on page 99.

It was decided that the interviews would solely be used to elicit the reasoning behind the responses made to both tests. The important items from the primary interviews on angle definition and recognition of a right angle would not be omitted as they were to be included in the Angles test.

6.3 ADMINISTRATION

The two tests were administered to all first year pupils (aged 11 and 12) during their first term at my school.

It is a London comprehensive school of 900 pupils with a mixed ability intake but a low proportion of pupils of high IQ. There is a large number of immigrant pupils, some of whom experience language difficulties. The five mixed ability first year classes are consciously composed of the same numbers of children in each IQ range, including remedial pupils. Significant differences in the test results of each form class would therefore not be expected. The classes each

TABLE 2

SOURCE OF SECONDARY TEST ITEMS

PRIMARY TEST ITEMS AND INTERVIEW

SECONDARY TEST ITEMS			1	2	3	4	5	5i	5ii	5iii	5iv	5v	5vi	6	7	8	9	10	11	12	INT
		1	A																		
		2		A																	
		3a																			A
		3b			I																
		3c														I					
	A	4												I							
	N	5															I				
	G	6				A															
	L	6i																			A
	E	6ii						I													
	S	6iii							I												
		6iv								I											
		6v									I										
		6vi										I									
		6vii					A														
		6viii																			A
		7																			A
MEASURING		1																			A
	A	2																			A
	N	3a																			A
	G	3b																			A
	L	3c																			A
	E	3d																			A
	S	3e																			A
		3f																			A

Key

I represents IDENTICAL

A represents ADAPTED

contain approximately 22 pupils of which just under half are girls. The form classes are named 1B, 1E, 1G, 1L and 1P after the initial letter of the surname of their form tutor. The pupils are taught mathematics in their mixed ability form classes.

Both tests were handwritten in two colours of banda on single A4 sheets upon which the pupils also wrote their answers. They appear in their original format on pages 18 and 19 of the Appendix. The arcs at each vertex were drawn in red as were the dotted lines on which pupils were supposed to answer the questions.

The tests were administered to each first year class by their normal mathematics teacher during mathematics lesson time. I was present at two of the test sessions. Pupils were informed that the tests were not judgments on them but were to ascertain what they had learned and remembered about angles in their primary school. Accordingly they were told to omit items on any topic which they had not previously met. They were allowed as much time as they required to complete the test.

As the Angles test was intended to test angle concepts rather than measuring, the use of a protractor was not permitted. No aids for the visual comparison of angles, such as tracing paper, were allowed.

For the Measuring Angles test classes 1B, 1E and 1G were given a full rotatable protractor (illustrated in Figure 2 on page 48) and classes 1L and 1P were given a half protractor. This enabled proficiency in measuring with these instruments to be compared.

Each interview consisted of working through both test papers which the pupil had previously completed and asking why certain responses had been given. The interviews were tape recorded but, due to problems with the recorder, some of them were lost. The remaining ones were transcribed for later analysis.

6.4 HYPOTHESES TESTED AND STATISTICAL TESTS USED

One month prior to taking the test, one first year class, 1G, had been taught the old unit on angle (Appendix, page 1) using a full rotatable protractor. This unit teaches the measuring and drawing of all types of angle, intuitive ordering of angles according to size and classification of acute, obtuse and reflex angles. It was expected that some parts of this unit would form a basis for the construction of the new unit. To test whether an improvement in understanding of the concept of angle and proficiency in measurement had ensued as a result of the teaching of this unit the test results of 1G were to be compared with those of the other classes, none of whom had met angle in secondary school.

For the Angles test it was intended that the results of 1G would be compared with those of 1B, 1E, 1L and 1P (referred to as 1BELP) to test the null hypothesis (NH1):

> The teaching of the angles unit produces no improvement in test results.

For each item of the Angles test except 3(a) the χ^2 test was used to test the null hypothesis NH1. Item 3(a) was not susceptible to statistical analysis as there was no valid criterion for judging whether responses were correct. 2×2 contingency tables were drawn up of the observed frequency of right (R) and wrong (W) responses of both 1G and 1BELP. There is only one degree of freedom for this 2×2 contingency table. No correction for continuity was made although this would have slightly reduced the obtained values of χ^2.

χ^2 is usually calculated from the formula:

$$\chi^2 = \sum \frac{(f_o - f_e)^2}{f_e} \qquad (6.4.1)$$

where f_o is the observed frequency of each outcome and f_e its expected frequency.

In the 2×2 case an alternative formula which does not involve expected frequencies exists. It is [20, page 17]:

$$\chi^2 = \frac{N(ad - bc)^2}{(a+b)(a+c)(b+d)(c+d)} \quad \text{with 1 d.f.} \qquad (6.4.2)$$

where a,b,c and d are the four observed frequencies respectively and $N = a+b+c+d$ is the sample size. This formula was used to calculate the values of χ^2 for each item. In no case was any expected frequency small enough to invalidate the test.

With one degree of freedom the critical value of the χ^2 distribution is 3.841 at the 5% level. If $\chi^2 > 3.841$ the null hypothesis is rejected at the 5% level as it is very unlikely that the observed differences would have been obtained

by chance. It needs to be remembered that, when a large number of tests are performed, some will appear to be significant anyway.

A table of the four observed frequencies and the value of χ^2 for each item are given together with the item itself before it is discussed in section 6.5.1.

In addition to the testing of the null hypothesis NH1 for each item in the Angles test a comparison of the overall performance of 1G and 1BELP was desired. It was therefore decided to use a t test on the mean number of correct responses per pupil in classes 1G and 1BELP to test the null hypothesis (NH2):

> There is no difference in the mean number of correct responses per pupil in each class.

The value of t was calculated from the formula:

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sigma\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}} \qquad \text{where } \sigma = \sqrt{\frac{N_1 S_1^{\,2} + N_2 S_2^{\,2}}{N_1 + N_2 - 2}} \; . \qquad (6.4.3)$$

$\bar{X}_1$ and $\bar{X}_2$ and S_1 and S_2 are respectively the mean and standard deviation of the number of correct responses per pupil in each class. N_1 and N_2 are the numbers of pupils in each class.

There are 106 degrees of freedom. Due to the small difference in the tabulated values for 60 and 120 degrees of freedom, the values for 60 degrees of freedom were used. Consequently, for a two-tailed test, if $t > 2.000$ the null hypothesis is rejected at the 5% level.

The value of t is given before the discussion of the test items in section 6.5.1.

For the Measuring Angles test 1G were issued with full rotatable protractors which they had been taught to use in the angles unit. Classes 1B and 1E (referred to as 1BE) were also issued with the full protractors while classes 1L and 1P (referred to as 1LP) were issued with a half protractor. It was intended that the results of 1LP would be compared with those of 1BE to test the null hypothesis (NH3):

> There is no difference in the proficiency in measuring angles with a half protractor which has been used before and with a full rotatable protractor which has not been used before.

It was also intended that the results of 1LP would be compared with those of 1G to test the null hypothesis (NH4):

> There is no difference in the proficiency in measuring angles with a half protractor and with a full rotatable protractor, both of which have been used before.

For each part of item 3 in the Measuring Angles test the χ^2 test was used to test the null hypothesis NH3 and the null hypothesis NH4. Calculations of χ^2 were made in the way described for the Angles test. In no case was any expected frequency small enough to invalidate the test.

A table showing the observed frequencies of right (R) and wrong (W) responses of 1G, 1LP and 1BE together with the χ^2 value for both statistical tests is given for each part of item 3 before it is discussed in section 6.6.1.

The t test was also used to test the null hypothesis NH2 for the responses to item 3 in the Measuring Angles test of classes 1LP and 1BE and of classes 1LP and 1G. There were respectively 85 and 63 degrees of freedom and consequently

the values tabulated for 60 degrees of freedom were again used.

The values of t are given before the discussion of the test items in section 6.6.1.

6.5 THE ANGLES TEST

The source of items and mode of presentation of these and of the responses is explained. Each item is then described and the responses to it are analysed. Tables of results are given. Shortcomings of the test considered to affect the interpretation of the responses are outlined. The results of classes 1G and 1BELP are compared and the results of the statistical tests are discussed.

6.5.1 THE TEST ITEMS - OBJECTIVES, DESCRIPTION, RESULTS AND INTERPRETATION

The items in the Angles test were mainly adaptations of those of the primary test with the addition of two items from the primary interviews. The source of each item, and whether or not it has been adapted, is indicated in Table 2 on page 99 . Item selection, rewording and variation of the form of presentation resulted from the analysis of both the primary test and interviews.

The two items adapted from the test were those relating to the recognition of a right angle and the definition of an angle. In the interviews the pupils had quickly understood the question being asked of them and consequently it was felt that the requirements could be made sufficiently clear in writing.

The items based on those of the primary test were: size comparison of acute, obtuse and reflex angles; recognition of equal angles including right angles in various

orientations; naming a right angle; stating the number of degrees in a full turn and in a right angle; approximating angle sizes in degrees.

The responses of each pupil to each item are shown, grouped according to form classes, in Tables 3 to 7 on pages 131 to 135. The results of the twenty one pupils in 1G are condensed in Table 8 on page 136 and those of the eighty seven pupils in 1BELP in Table 9 on page 137. Tables 3 to 7 also indicate the number of correct responses given by each pupil. Tables 8 and 9 indicate the number of acceptable responses to each item, the percentage facility of each item, the number of pupils omitting each item and the frequency of the most common errors.

Each item is reproduced below together with the correct answer. A table of the frequency of right (R) responses and wrong (W) responses for classes 1G and 1BELP is given, as is the value of χ^2 (significant at the 5% level if $\chi^2 > 3.841$). The objectives tested and details of presentation are described. Noteworthy responses are outlined and interpretations made.

In considering the null hypothesis NH1 (see page 101) being tested the discussion following some items distinguishes between the responses of 1G and 1BELP pupils. Due to the small size of class 1G the discussion on certain items considers aggregate responses from all classes. Pupils are referred to as bright, average or weak depending upon whether the total number of correct responses they have given is above average, average or below average for their class.

The full test in its original format is in the Appendix on page 18.

$t = 2.902$ and thus the null hypothesis NH2 (see page 103) is rejected at the 5% level.

Item 1

Which is the LARGEST of these angles?

(a) (b) (c) (d)

	R	W	
1G	18	3	Answer: (c)
1BELP	64	23	$\chi^2 = 1.366$

The objective was to test if pupils were able to compare intuitively the size of both acute and obtuse angles, with arms of the same length but in different orientations.

The angles were drawn as clearly different in size.

In all eight pupils omitted this item. Because most of these pupils attempted the following harder item it appears that the omissions were due to the layout in which the answer space was too near the 'Name' and 'Form' spaces. (See Appendix, page 18). Six of the pupils who omitted the item selected (c) for item 2 and it is not clear to which item this was intended to be the response.

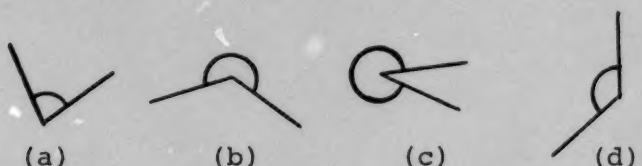
The most frequent error, and the only error from a pupil in 1G, was (b). Twelve pupils in 1BELP, most of whom were weak, and one in 1G selected it. This error was probably due to the difficulty which the pupils experienced in visually overlaying (b) and (c) in their mind.

Two weak pupils in 1P were observed using a ruler to measure the distance between the ends of the arms. They answered (b) although this distance is greater in (c).

Pupils in 1G were more certain of the criteria for judging angle size as none of them opted for other distractors whereas five pupils from 1BELP did so. Thus the weaker pupils in 1G had not opted for illogical distractors which indicated that they had benefitted from the angles unit.

Item 2

Which is the LARGEST of these angles?



(a) (b) (c) (d)

	R	W	
1G	16	5	Answer: (c)
1BELP	48	39	$\chi^2 = 3.095$

The objective was to test if pupils were able to compare intuitively the size of acute, obtuse and reflex angles with equal length arms but different orientations.

The angles were drawn as clearly different in size. The sum of angles (b) and (d) was approximately 360° and hence their arms were spread apart by similar amounts.

It was expected that pupils with a static perception of angle as the sharpness of a point would select (d) as they would be unable to extend this notion to reflex angles.

The five errors made by pupils of 1G were spread almost equally among all distractors. Of the thirty-six errors made by pupils in 1BELP thirty were (d) and were made by pupils of various abilities.

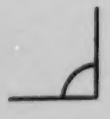
Most pupils in class 1G have clearly moved beyond the static criterion of sharpness which is only applicable to

acute and obtuse angles, whether or not they are able to perceive the rotations represented by the arcs.

Item 3

This item consisted of three parts relating to a right angle, the name of which was not given. A drawing was provided in which the arms were positioned parallel to the sides of the page in a familiar orientation. The customary right angle symbol was omitted in order to test the ability to recognise the right angle by its shape alone.

(a) Write down one place in the classroom where you can see an angle like this.



The objective was to test if pupils understood the concept of a right angle.

The responses were frequently vague, the most common objects listed being walls, windows, doors, the blackboard and the ceiling. Pupils often did not specify the particular part of this object where the right angle could be observed but it seemed from questions which they asked about this item during the tests that most of them did in fact know this. About one third of all pupils used the word 'corner' in their responses but it was deemed that the inclusion of this word in a response was not a fair criterion for judging correctness and hence the question was considered to be invalid. The responses to it were not included in tables 3 to 9 or in any statistical analysis.

(b) What is the special name for this angle?

	R	W	
1G	18	3	Answer: a right angle
1BELP	54	33	$\chi^2 = 4.256$
			sig. at 5% level

The objective was to test if pupils could recall the name 'right angle'.

Three bright pupils from 1BELP answered 'right angled triangle' and one weak pupil 'right triangle' illustrating the interference between this concept and that of a right angle when work with triangles is introduced prematurely. Three pupils from 1BELP chose 'acute' indicating the readiness to apply a memorised label without considering the criteria for its use. Two weak 1BELP pupils wrote '90 degrees angle' showing that this name was perceived qualitatively rather than as indicating the measure of the angle. All of these responses were classed as incorrect.

Pupils in 1G achieved significantly better results than those in 1BELP indicating that frequent revision or use of the name is required if it is to be recalled.

(c) How many degrees are there in this angle?

	R	W	
1G	14	7	Answer: 90
1BELP	50	37	$\chi^2 = 0.592$

The objective was to ascertain if pupils knew the degree measure of a right angle represented as a static ray pair.

The errors did not fall into a particular pattern but five pupils, including two from 1G, answered 2 and one answered 1 indicating the continued use of the half right angle and right angle as units.

A confusion with degrees as a measure of temperature was revealed in two responses of $90^{\circ}C$. A confusion between

the symbol for degrees and percentages was exhibited by three responses using the % sign. Responses of 90°C and 90% were classed as correct.

This item was poorly answered by both 1G and 1BELP indicating that greater emphasis on it is required in the new angles unit than was given in the old one.

Item 4

How many degrees are there in ONE FULL TURN?

	R	W	
1G	12	9	Answer: 360
1BELP	38	49	$\chi^2 = 1.234$

The objective was to ascertain if pupils knew or could calculate the degree measure of this specific angle, considered as a rotation.

This item was constructed to contrast with item 3(c) in which the degree measure of a static ray pair was requested. It was presented following item 3(c) to prompt pupils to calculate 4×90 if they could not recall the answer or had not met it before.

It was expected that a number of pupils would answer 180. This can arise because 180 is the largest number seen on a half protractor and can therefore be assumed to be the largest possible measure of an angle. There are also geometric and linguistic reasons for thinking that there are 180° in a full turn even when the magnitude of 180° is clearly understood. These points are discussed fully in the interpretation of responses to item 6 of the primary test on page 38.

Thirteen pupils from 1BELP and two from 1G answered 180. These 1G pupils were weak on all items involving degrees. Of the thirteen 1BELP pupils ten had correctly stated that there were 90° in a right angle yet had clearly been unable to utilise this in calculating the number of degrees in a full turn in the manner anticipated. The response of these ten pupils indicated a confusion between the notions of a full and a half turn rather than the inability to multiply 90 by 4. The fact that only six of the thirteen 1BELP pupils made any reasonable attempt at an angle estimation item while the other seven did not answer sensibly in relation to either 180 or 360 as the degree measure of a full turn indicated that their response could also be due to their incorrect recall of what was to them an isolated and meaningless fact.

Item 5

ABOUT how many degrees are there in each of these angles?

The objective was to test if pupils could apply their knowledge of degree measure.

This item consisted of five parts in which the size of a variety of angles in different orientations was to be estimated.

A range of answers was acceptable for each part. These were not chosen symmetrically about the accurate size of the angle but were intended to include values which were likely to be chosen, such as multiples of 5, and which approximate

sufficiently closely to the correct value. In no case did a response fall one or two degrees outside the limit of these ranges which was nearer to the true value.

The statistics given for each part of the item relate to the numbers of acceptable rather than accurate responses. One pupil from 1G and twenty-four from 1BELP omitted the whole item.

(a)

Answer:	45		R	W
Acceptable:	40 - 50	1G	7	14
χ^2 = 0.558		1BELP	22	65

This angle was presented in a familiar orientation with one arm vertical. It was intended to be the easiest part of this item because of its orientation and because of the large amount of work on right angles and half right angles which pupils had met in primary school.

Most pupils who attempted this part were able to suggest an answer between 0 and 90 but were very inaccurate in so doing. There was a wide variety of errors, the most common being 30 (fifteen pupils), 20 (ten pupils) and 60 (six pupils). Altogether fifty of the eighty-one responses were fairly reasonable, being from 30 to 60.

The facility in this part is very low, but surprisingly so for 1G.

(b)

Answer:	78		R	W
Acceptable:	70 - 85	1G	12	9
χ^2 = 6.061		1BELP	25	62
sig. at 2% level				

This angle was presented in a familiar orientation with one arm vertical. It was intended to be easy to estimate accurately as it was close to 90°.

The pupils, especially those in 1G, found it easier to estimate more accurately than part (a). Altogether forty-seven of the eight responses were fairly reasonable, ranging from 60 to 90. One response of 95 and two of 110 were given by 1BELP pupils who had correctly stated the number of degrees in a right angle in item 3(c). They have added, rather than subtracted, a small increment to 90.

This was the part of the item in which the highest facility was achieved for both 1G and 1BELP. The pupils in 1G achieved significantly better results than those in 1BELP.

(c)

Answer:	158		R	W
Acceptable:	150 - 170	1G	5	16
$\chi^2 = 0.484$		1BELP	15	72

This angle was presented in an unfamiliar orientation. It was drawn as fairly near to 180° in order that an obtuse angle could be tested without the interference caused by difficult interpolations between 90° and 180°.

This was the part of the item of lowest facility for 1G. The one pupil who omitted it had omitted the whole item. Four other pupils chose values in the range 25 to 75. The remaining sixteen pupils, who chose values in the range 100 to 175, probably possessed a notion of the degree measure of obtuse angles but only five of them were able to estimate sufficiently accurately and seven of them answered from 100 to 120.

1BELP pupils also performed very poorly with only thirty pupils choosing values in the range 100 to 175 of whom twenty one chose values from 100 to 120. A further eight pupils selected 180.

The choice of responses in the range 100 to 120 is attributable to the inability to estimate numbers between 90 and 180. Questions put to pupils who were seen to make these errors revealed that they were aware that the angle was larger than 90° and therefore suggested something a little larger than this. They did not look at how close the angle was to 180° or consider whether it was nearer to 90° or to 180°. Nine of the 1BELP pupils and six of the 1G pupils who had answered in this range were of above average ability and had answered a number of other items involving degrees correctly.

Of the eight pupils who selected 180, five had given the number of degrees in a right angle and a full turn correctly. They should therefore have been aware that a straight angle was 180° and that the angle in this part was clearly less than a straight angle. In fact 180° is a good approximation to the size of angle (c) and it is acceptable to interpret the question 'ABOUT how many degrees ... ?' as a request for an approximation rather than an estimation to be made. Two of these five pupils later gave answers of 360 for (e) showing that their interpretation of the question was definitely as an approximation. However a pupil's inability to estimate when dealing with numbers may also result in approximations being given.

In all ten pupils from 1BELP indicated that they had made an approximation by answering one or two of parts (b), (c) and (e) as respectively 90, 180 and 360. In 1G two pupils answered (b) as 90 and one answered (e) as 360 but none answered (c) as 180. These three pupils made many other uninterpretable responses to item 5 which indicated that they were not attempting to give approximate answers.

Even though the angles unit had not included a section on estimation (it had been asked as part of one homework question) the pupils in 1G did not interpret the questions as approximations in the same way that some pupils in 1BELP had. The 1G pupils were far more aware of the range of possible degree measures of obtuse angles but many of them were unable to estimate accurately between 90 and 180 indicating a lack of primary school experience of estimation of large numbers.

(d)

Answer: 267

Acceptable: 260-280

$\chi^2 = 6.083$

sig. at 2% level

	R	W
1G	7	14
1BELP	10	77

This reflex angle was presented in an oblique orientation. It was chosen to be approximately 270° to check if this could be recognised or found by the subtraction of 90° from 360° rather than by a difficult interpolation between 180° and 360°.

It was expected that pupils unfamiliar with reflex angles would give responses of approximately 90.

Three pupils from 1G and twenty from 1BELP gave answers near to 90. The number of very accurate estimations

of the 93° angle was surprising in that nine of them were provided by pupils who did not answer in the range 70 to 90 for part (b).

This part was omitted by more pupils from both classes than any other part. Of the four pupils who omitted this part alone, the one in 1G answered part (e) correctly while the others had offered reasonable attempts at acute and obtuse angles. It seems that they had all realised that an answer near 90 was inappropriate and had therefore not suggested one and yet they were unable to deal with estimating large numbers in the way they had done with smaller numbers.

For pupils of 1BELP this was the part of the item of lowest facility. They clearly did not use the method of subtracting 90 from 360 and, even those who indicated from their response to part (e) that they could recognise reflex angles, were unable to interpolate sensibly between 180 and 360.

The pupils in 1G achieved significantly higher results than those of 1BELP not only because they had met reflex angles but because they were familiar with the angle of 270°. It has been seen that they were unable to interpolate well in part (c) and thus it can be concluded that part (d) was answered either through recognition or subtraction of 90 from 360.

(e)

Answer: 343

Acceptable: 330-355

$\chi^2 = 19.974$

sig. at 0.1% level

	R	W
1G	10	11
1BELP	7	80

This reflex angle was presented with its vertex uppermost, in a very unusual orientation. It was chosen to be only a little less than 360° to enable all pupils who could recognise this fact to answer correctly without having to perform a difficult subtraction or interpolation between 180° and 360°.

It was expected that pupils unfamiliar with reflex angles would give responses of approximately 20.

One pupil from 1G and eighteen from 1BELP attempted to estimate the acute angle.

A number of pupils recognised the approximate magnitude of the angle. In class 1G thirteen pupils answered in the range 300 to 370. In class 1BELP eighteen pupils answered in the range 300 to 380; thirteen of these having correctly stated that there were 360° in a full turn.

Both classes achieved higher facility in this part than in both part (d) and part (c), the obtuse angle. This indicated that pupils found it far easier to estimate small numbers, either as measures of acute angles or to be subtracted from 360 to obtain the measure of a reflex angle between 270° and 360°. Both obtuse angles and reflex angles between 180° and 270° were found harder to estimate, which revealed that pupils had been given insufficient experience in the estimation of numbers over 100. The overall difficulty in estimating mid-values was highlighted by the lower facility of part (a) than part (b) for both clases.

Pupils in class 1G achieved significantly better results in part (e) than the pupils in 1BELP. This was due to their familiarity with reflex angles.

In both reflex angle parts the facility for 1G pupils was fairly low indicating that the angles unit did not present this concept sufficiently well.

Item 6

Which angle is BIGGER or are they ABOUT THE SAME SIZE? Answer this question for each pair of angles.

This item consisted of eight parts and was intended to highlight the importance ascribed to various attributes of angles when comparing their size.

The use of the wording 'ABOUT ... ' was to encourage pupils only to select an angle as bigger if it was clearly much bigger.

In each part, except the first, the radius of the two arcs was identical.

The omission of the worked example from this item, which had been provided in item 5 of the primary test, resulted in some unclear responses. Four pupils from 1G and eleven from 1BELP answered some parts as 'bigger' without stating any angle name while two pupils from 1P answered each part with 'yes' or 'no'. All of these responses were classed as incorrect.

Three pupils from 1G and fourteen from 1BELP omitted the whole item.

(a) (b)

Answer: same

$\chi^2 = 4.072$

sig. at 5% level

	R	W
1G	12	9
1BELP	29	58

The objective of this part was to test if pupils looked simply at the length of arc, irrespective of its radius.

Angles (a) and (b) were equal angles drawn in the same orientation with one arm of 2 cm and the other of 1.9 cm but marked with arcs of different radius.

It was expected that (b) would be selected by pupils who compared arc lengths but not 'sharpness' or who gave more importance to the former than to the latter. Pupils selecting the larger arc would not be relating the arc to the rotation it represented.

Five pupils from 1G and thirty-three from 1BELP selected (b).

The 1G pupils achieved significantly better results than the 1BELP pupils. They may have considered an attribute such as sharpness in comparing the angles but they were more aware than pupils from 1BELP of the equality of the rotations represented by the arcs.

(c)

(d)

Answer: same

$\chi^2 = 5.214$

sig. at 2.5% level

	R	W
1G	14	7
1BELP	34	53

The objective was to test if importance was ascribed to arm length in comparing the size of angles in different orientations. This part specifically tested whether the length of one arm of an angle was considered to affect its size.

The angles were presented as slightly separated vertically opposite angles because these often look intuitively equal. Both arms of (d) were equal in length to the shorter

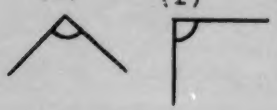
arm of (c).

It was expected that (c) would be selected by pupils who felt that angle size was related to arm length.

The majority of pupils making incorrect responses selected (c). Twenty seven from 1BELP and three from 1G did so. Of these 1BELP pupils nineteen later opted for (g) which had longer arms than (h). The three 1G pupils, however, later opted for (h), the vertex of which was higher than that of (g). Only three pupils, all of whom were from 1BELP, opted for (d).

The results achieved by 1G were significantly better than those of 1BELP. 1G pupils were more aware than 1BELP pupils that arm length did not affect angle size.

(e) (f)



Answer: same

$\chi^2 = 0.187$

	R	W
1G	11	10
1BELP	41	46

The objective was to test if a right angle could be recognised when drawn in an unfamiliar orientation.

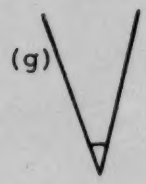
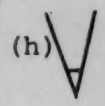
The angles were both drawn with arms of 1.3 cm. The arms of (f) were square to the page but not in the most familiar first quadrant. The customary right angle symbol was not used in order to test the ability to recognise a right angle by its shape alone.

It was expected that (e) might be selected by those who felt that the horizontal distance apart of the end of its arms looked large and concentrated on this rather than on the vertex of the angle. (f) could be selected by those who felt that its vertical arm was longer than those of (e)

as it stretched farther downwards, the vertices of both angles being alongside each other.

In 1G five pupils selected (e) and none (f). In 1BELP eight selected (e) and fifteen selected (f), of whom eight also selected other angles in this item with longer arms, (c) and (g).

The low, and similar, facilities for both classes on this item illustrated the difficulty experienced in the visual overlaying of angles in orientations oblique to each other even in the particular case of the frequently met right angle. The work in the angles unit had not improved pupils' ability to recognise a right angle.

(g)  (h) 

Answer: same

$\chi^2 = 3.58$

	R	W
1G	14	7
1BELP	38	49

The objective was to test if importance was ascribed to arm length in comparing the size of angles, each of which had equal length arms and was drawn in the same orientation. This part specifically tested whether the length of both arms of an angle was considered to affect its size.

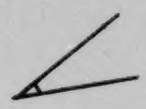
The arms of angle (g) were 2.5 cm in length and those of angle (h) were 1.3 cm in length. Both angles were oriented identically, although not symmetrically about the vertical. The vertex of (h) was higher than that of (g).

It was expected that (g) would be selected by pupils who felt that angle size was related to arm length and (b) by those who felt that it was affected by position.

In 1G three pupils selected (h) and one selected (g). In 1BELP five pupils selected (h) and twenty one selected (g). Of the eight pupils selecting (h) five had previously selected both (c), which is drawn above (d), and (e), the arms of which end above those of (f). Of the twenty-one 1BELP pupils who selected (g), nineteen had previously selected (c), an angle with one arm longer than those of (d).

The pupils of 1G achieved better results than those of 1BELP, although not significantly so. The errors discussed indicated a large degree of consistency in the criteria used fro comparing angle sizes.

(i)

(j) 

Answer: (i)

$\chi^2 = 0.665$

	R	W
1G	11	10
1BELP	37	50

The objective was to test if a reflex angle could be recognised as larger than the acute angle, which when added to it would make 360°.

Angles (i) and (j) were drawn in the same orientation with (i) above (j) and with arms of length 1.9 cm.

It was expected that those pupils who considered only the relative position of the two arms of each angle would judge (i) and (j) to be equal, as would those pupils who could not conceive of a reflex angle. It was expected that the correct answer (i) would be selected by those pupils who had previously met reflex angles and by those who compared arc length rather than the relative position of the two arms. Comparison of arc length could be the criterion used only because it was the main clear difference between the angles

(the other being the positioning of (i) above (j)) or because the pupil was aware of the rotation represented by the arc.

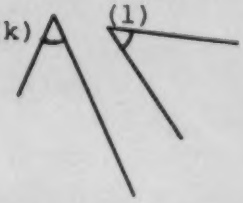
Four pupils in 1G and seventeen in 1BELP thought that (i) and (j) were equal.

Of the thirty three 1BELP pupils who had selected (b), which had a larger loop than (a), twenty three also selected (i). All five 1G pupils who had selected (b) also selected (i). Of the five 1BELP pupils who had selected (h), the vertex of which was above that of (g), four also selected (i). Of the three 1G pupils who had selected (h), two selected (i).

It can be seen from this comparison of responses that (i) can have been selected for the wrong reasons and thus cause an apparent increase in facilities. Consequently it can be seen that the objective of this part has not been well tested.

The low facility for 1G pupils indicated that their ability to recognise reflex angles had not been improved sufficiently by the angles unit. The low facility for 1BELP was expected as many of them had not previously met reflex angles. It showed that few of them perceived angle as a rotation or were able to extend this notion beyond their experience of acute and obtuse angles.

(k) (l)



Answer: same

$\chi^2 = 1.458$

	R	W
1G	12	9
1BELP	37	50

The objective was to test if importance was ascribed to arm length in comparing the size of angles in different orientations. This part specifically tested whether a short

and a long arm were considered to render the size of an angle larger than one with two middle sized arms.

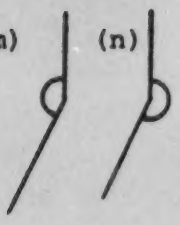
Angles (k) and (l) were oriented differently but both with their vertices upwards. (l) had equal arms and (k) was drawn with one arm clearly shorter and one arm clearly longer than those of (l).

It was expected that pupils who felt that the length of the longest arm determined the size of the angle would select (k). Those pupils who felt that the short arm of (k) rendered the angle smaller were expected to select (l).

Five pupils from 1BELP and none from 1G selected (l). Twenty from 1BELP and three from 1G selected (k). Four of those who selected (l) had previously selected (c), one arm of which was longer than those of (d). Thirteen of those from 1BELP who had selected (k) had also selected both (c) and (g), the latter of which had both arms longer than those of (h). Fifteen of the 1BELP pupils and all of the 1G pupils who selected (k) or (l) had also been unable to recognise angles (e) and (f), which were in differing orientations, as equal.

The low facility for 1G pupils on this part indicated the need for more comparison of angles in different orientations in the angles unit. The comparison of errors with those in other parts of this item indicated a nucleus of pupils who considered arm length to affect angle size.

(m) (n)



Answer: (n)

$\chi^2 = 0.106$

	R	W
1G	3	18
1BELP	15	72

The objective was to test if a reflex angle could be recognised as larger than the acute angle which when added to it would make 360°.

Angles (m) and (n) were drawn in the same orientation but with the lower arm of (m) slightly longer than that of (n).

It was expected that those pupils who considered only the relative position of the two arms of each angle would judge (m) and (n) to be equal, as would those pupils who could not conceive of a reflex angle. It was expected that the correct answer (n) would be selected by those pupils who had previously met reflex angles and by those who compared arc length rather than the relative position of the two arms. Comparison of arc length could be the criterion used only because it was the main clear difference between the angles (the other being the slightly longer lower arm of the smaller angle (m)) or because the pupil was aware of the rotation represented by the arc.

Ten pupils in 1BELP and one in 1G opted for (m). Of these all except two members of 1BELP had previously selected (c) or (k) or both, angles which also possessed one longer arm.

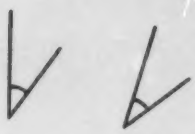
Twelve pupils in 1G judged the angles to be equal. Of these, four had also found (i) and (j) equal while six had correctly selected (i) as larger than (j). Forty pupils in 1BELP judged the angles to be equal. Of these, thirteen

had also found (i) and (j) equal while sixteen had correctly selected (i) as larger than (j).

The extremely low facilities for this part (14% for 1G and 17% for 1BELP) indicated the great difficulty in recognising reflex angles between 180° and 270°. It was the only part of any item in both tests in which 1BELP achieved a higher facility than 1G. The lack of correlation of error selection with those in part (i) (j) suggested that this part may have only been glanced at briefly. The optical illusion effect caused by the parallel sets of arms with loops on their outsides could easily cause the angles to be judged as equal before any close examination had taken place. The exact format of this part was considered to have resulted in the poor testing of the objective.

(o) (p) Answer: same

$\chi^2 = 2.369$



	R	W
1G	15	6
1BELP	46	41

The objective was to test if importance was ascribed to the orientation of an angle in determining its size.

Angles (o) and (p) were each drawn with their left arm of length 1.6 cm and right arm of length 1.3 cm. The left arm of angle (o) was vertical while that of angle (p) was 15° from the vertical.

It was expected that (p) would be selected by those pupils who perceived angles as 'clock hands', judging their size by the angle between the right arm and the vertical in a similar manner to the determination of a bearing.

In class 1G no pupils selected (o) and two selected (p), both of whom had previously selected (c), (e) and (h), angles which are higher on the page.

In 1BELP eight pupils selected (o) and eight selected (p). All of those who selected (o) had previously selected (c), six of them (b), five of them (g), five of them (i) and five (k), angles which had longer arms or longer loops. Of those who selected (p), five had selected each of (b), (c), (f), (g), (i), (k) and (m), angles which had longer arms or longer loops. Three of those who selected (o) had correctly answered more than the average number (7.4) of items, as had only one of those who selected (p).

The facility for both classes was high with pupils in 1G achieving better results than those in 1BELP, but not significantly so. The objective was not fully tested by this part in which the orientation of the angles differed by only 15^{o}. A series of items is required in which the differences in orientations are varied.

Item 7

Which ONE of these is the best way to describe an angle between two lines?

(a) The amount of space between the lines.
(b) The distance between the lines.
(c) The length of the lines.
(d) The amount of turn from one line to the other.
(e) The direction the point faces.

	R	W	Answer: (d)
1G	11	10	$\chi^2 = 2.991$
1BELP	28	59	

The objective was to test if angle was perceived as a rotation and to elicit the concepts of angle most commonly possessed.

The question included the wording 'between two lines' in order that the alternatives could be expressed in simpler, more concrete terms. The correct alternative was a dynamic definition of angle. None of the alternatives gave a correctly worded static definition of angle.

Alternative (a) was worded loosely and intended to be selected by pupils with an area notion of angle. Alternative (b) was intended for pupils who measure the distance between the ends of the arms. Alternative (c) was intended for pupils who select angles with longer arms as larger, such as those who opted for (c), (g) or (k) in item 6. Alternative (e) was intended for pupils who felt that orientation distinguished angles.

In class 1G six pupils chose (a), none (b), one (c) and two (e). In class 1BELP eleven chose (a), twenty two (b), four (c) and nine (e). These responses did not follow any particular pattern in relation to the same pupil's responses to item 6.

The large number of 1BELP pupils selecting (a) and (b) reflects their static view of angle. The low facility for 1G indicates that more discussion on the nature of angle is required in the angles unit.

6.5.2 TABLES OF RESULTS

Tables of each pupil's responses to each test item are given, grouped according to form classes. The condensed results for each item for classes 1G and 1BELP are also tabulated.

TABLE 3

EACH PUPIL'S RESPONSES TO THE ANGLES TEST ITEMS: CLASS 1G

Correct responses are in RED.

ITEM NUMBER

	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7	
CORRECT	C	C	ra	90	360	45	78	158	267	343	s	s	s	s	i	s	n	s	d	19
PUPILS																				PUPIL'S SCORE (mean 10·6)
Kerry	C	C	ra	90	360	45	90	110	200	300	S	S	S	S	i	s	big	s	d	14
Tina	C	C	ra	90	360	30	85	170	127	350									a	8
Linda	C	C	ra	90	360	30	85	170	127	350									a	8
Kastur	C	C	ra	90	360	45	85	175	270	355	S	S	S	S	S	S	S	S	a	15
Andrey	C	C	ra	80	160	40	85	110	175	150	big	S	S	S	big	S	S	S	d	11
Hema	C	C	ra	90	360	15	85	120		355	b	S	e	g	i	S	S	S	d	12
Shaata		C	ra		360	45	85	75	68	510									a	5
Susana	C	B	ra	80							b	c	e	h	i	k	m	p	d	4
Sunita		C	right	2	165	60	90	125	126	135	S	big	big	S	S	big	S	S	d	5
Mustapha	C	D	ra		380	12	14	25		29	S	S		S	S		S	S	d	7
Peter	C	C	ra	90	380	30	80	160	90	370	S	S	S	S	i	S	n	S		14
Ibrar	C	C	corner	2	180	130	140	160	180	360	S	S	S	S	S	S	S	S	a	9
Jason	C	C	ra	90	360	20	30	160	270	340	b	S	e	S	i	k	S	S	c	12
Steven	C	A	ra	90	380	20	40	100	140	225	S	c	e	h	j	S	S	p	a	6
Deepak	C	C	ra	90	360	45	60	110	270	380	S	S	S	S	i	S	n	S	d	17
Jay	C	C	ra	90	1080	20	85	130	220	340	S	S	S	S	i	S	S	S	d	15
Patrick	C	C	ra	90	360	30	70	110	270	350	S	S	S	S	big	big	big	big	d	13
Shaun	C	C	ra	90	360	50	80	120	270	350	S	S	S	S	i	S	n	S	d	18
Harish	B	B	ra	90	380	30	85	50	85	80	b	S	S	S	i	S	S	S	e	9
Mark	C	A	vertical angle	70	180	35	20	60	90	50	b	c	e	h	i	k	S	S	e	3
Paul	C	C	ra	90	360	45	85	140	220	340	S	S	S	S	i	S	S	S	d	17
	21																			
ACCEPTABLE RESPONSES	18	16	18	14	12	7	12	5	7	10	12	14	11	14	11	12	3	15	11	222
% FACILITY	86	76	86	67	57	33	57	24	33	48	57	67	52	67	52	57	14	71	52	

Key

- s represents the response 'SAME'
- big represents the response 'BIGGER'
- ra represents the response 'RIGHT ANGLE'
- represents an omission

Notes

The ranges of acceptable responses for item 5 are:

5a	40-50
5b	70-85
5c	150-170
5d	260-280
5e	330-355

TABLE 4

EACH PUPIL'S RESPONSES TO THE ANGES TEST ITEMS: CLASS 1B

Correct responses are in RED.

ITEM NUMBER

	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7	
CORRECT	C	C	ra	90	36	045	78	158	267	343	s	s	s	s	i	s	n	s	d	19
PUPILS																				PUPIL'S SCORE (mean 8·3)
Sadhana	C	C	ra	90	36	040	95	75		175	S	S	S	s	S	S		b		11
Amarjit	C	C	ra	4	8													b		3
Simone	C	C	ra	4	8							big						b		3
Angela	C	D	ra	90	36	040	80	180	90	20	S	S	S	g	s	S	s	o	a	11
Petruia	C	D	ra	90	180	40	10	170	60	90	S	S	S	S	s	k	s	S	d	11
Hala	C	D	ra	90	180	45	80	110	90	5	S	c	f	g	s	l	s	S	a	7
Vonessa	C	C	ra	90	180	45	80	170	205	10	S	S	S	S	i	s	S	S	d	15
Rachel	B	C	ra	40	90	30	35	20	10	20	b	s	e	h	i	s	s	s	b	6
Jeanette	C	C	ra	90	36	045	85	135	270	175	S	S	S	S	i	s	n	s	d	17
Richard	D	B	ra	90	360	45	85	170	90	15	S	S	S	S	i	l	s	S	d	13
Anthoney	C	C	ra	90	360	45	80	100	270	300	b	c	f	s	i	s	n	s	d	14
David	C	C	ra	90	180	45	90	180		15	b	c	f	g		k	s	S	e	6
Robert	A	A	ra	45	260	30	43	10	42	1	b	d	f	g	i	k	m	o	a	2
Jason	C	C	ra	90	360	15	25	75	105	135	S	big	S	S	big	s	s	big	b	9
Arron	C	D	ra	90	360	45	80	180	60	300	b	S	S	S	s	S	s	S	d	12
Jitendra	C	C	ra	90	360	45	80	120	280	350	b	S	f	S	i	k	s	S	d	14
Keith	C	D	ra	90	60	30	90	100	60	10	b	big	S	big	s	S	s	big	d	6
Antony	C	C		2	4						S	c	f	g	i	k	m	S	b	5
Mark	C	C									b	c	e	g	i	k	m	p	b	3
Roger	B	D		90	90														b	1
Timothy	C	B	ra	75	300	25	50	75	100	10	S	big	big	S	big	S	S	big	b	5
21																				
ACCEPTABLE RESPONSES	17	12	18	14	8	11	8	4	3	1	9	9	9	10	9	10	2	12	8	174
% FACILITY	81	57	86	67	38	52	38	19	14	5	43	43	43	48	43	48	10	57	38	

Key.

- S represents the response 'SAME'
- big represents the response 'BIGGER'
- ra represents the response 'RIGHT ANGLE'
- represents an omission

Notes

The ranges of acceptable responses for item 5 are:

5a	40-50
5b	70-85
5c	150-170
5d	260-280
5e	330-355

TABLE 5

EACH PUPIL'S RESPONSES TO THE ANGLES TEST ITEMS : CLASS 1E

Correct responses are in RED.

ITEM NUMBER

	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7	
CORRECT	C	C	ra	90	360	45	78	158	267	343	S	S	S	S	i	S	n	S	d	19
PUPILS																				PUPIL'S SCORE (mean 7·8)
Wendy	C	C	ra	90	360	20	80	160	270	340	S	S	S	S	i	S	n	S	d	18
Siobhan	C	C	ra	90	360	45	70	110	140	180	b	c	e	h	i	k	n	o	d	9
Bernadette	C	C	ra	90	360	10	10	20	30	30	b	S	e	S	i	S	S	S	d	11
Razia	B	D	ra	60	154						b	S	S	S	i	S	S	S	b	7
Shabana	C	D									b	c	e	h	i	l	m	p	d	3
Joanna	B	C			2															1
Antraulla	C	D	ra	45	180	20	40	60	55	10	b	c					S	S		3
Julie	C	D	ra	90	360	45	80	170	85	S	S	S	S	S	S	S	S	S	d	14
Tracy	B	C		90		30	50	70	80	90	big	S	big	S	big	S	S	S	c	6
Susan	C	C	ra	95		10	10	30	40	45	b	S	S	S	i	S	m	o	d	9
Barbara	C	C	aaaa	90																3
Peter	C	C		90	360														c	4
Robert	C	C	ra	90	180	20	15	45	90	180	b	S	S	S	i	S	S	S	d	11
Edmond	C	C		10	20	2	5	7	9	12	b	c	e	g	i	k	n	p		4
David A.W.	C	B	ra	90	360	45	60	110	90	20	S	S	S	S	j	S	S	S	d	12
Roger	D	C	ra	90	360	20	25		85	360	a	d	S	h	i	k	s	S	c	8
David G.	B	D	ra	90			40												d	3
Diego	C	D																	e	1
Alan	C	C	ra		360						S	big	S	S	S	S	S	S	d	10
Ray	B	A	ra	90	360	20	80	60	70	10	S	c	S	g	i	S	n	S	a	10
Sunil	C	D	ra	90	360	45	80	170	90	10	b	c	f	g	i	l	s		d	9
Saber	C	C	ra	90	360	30	80	180	220	340	S	S	S	S	i	S	n	S	b	15
	22																			
ACCEPTABLE RESPONSES	16	13	15	14	11	4	7	3	1	2	6	9	10	10	12	11	5	12	11	172
% FACILITY	73	59	68	64	50	18	32	14	5	9	27	41	45	45	55	50	23	55	50	

Key

- S represents the response 'SAME'
- big represents the response 'BIGGER'
- ra represents the response 'RIGHT ANGLE'
- represents an omission

Notes

The ranges of acceptable responses for item 5 are:

- 5a 40-50
- 5b 70-85
- 5c 150-170
- 5d 260-280
- 5e 330-355

TABLE 6

EACH PUPIL'S RESPONSES TO THE ANGLES TEST ITEMS: CLASS 1L

Correct responses are in RED.

ITEM NUMBER

	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7	
CORRECT	C	C	ra	90	360	45	72	158	267	343	S	S	S	S	i	s	n	s	d	19
PUPILS																				PUPIL'S SCORE (mean 6·8)
Fadhia	C	D	ra	90	360	60	80	110	94	8	b	c	S	g	s	k	n	S	e	8
Bosede			ra	6	360														b	2
Grace	C	C	ra			2	1	180	270	360	S	c	S	g	s	k	s	s		7
Shobhna	C	C	rat	90	360	45	65	180	270	360	b	c	e	S	i	s	s	s	a	10
Jackie	C	D	ra	60	360	12	11	114	127	30	S	c	S	g	j		m	p	b	5
Sheila	C	C	ra	90	180						b	c	S	g	j	S	s	S		7
Varsha		C	ra	90	360	70	100	150	90	10	S	S	S						d	9
Michelle	C	C									b	S	e	s	i	s	m	s	d	8
Vandana	C	C	ra	90	360	45	100	155	180	345	S	S	S	S	i	S	n	s	b	16
Louise		C	ra		360														b	3
Bridgit	C	D	ra	90	180	80	20	10	20	80	b	c	f	g	i	k	s	p		4
Luke	C	D			360	50	70	160	90	70	b	d	S	h					d	7
Stephen K.	B	C	rt.	2	3															1
Stephen	D	D									S	big	S	big	s	big	big	S		3
Bimal	C	D	ra	90	180	80	20	10	20	80	b	c	S	S	s	k	s	p	e	5
Sean	C	C	rat	90	360	45	65	170	300	350	S	S	f	S	i	S	n	s	d	15
Jayesh		C	rat	00	360	45	67	170	300	350	S	S	big	S	big	S	big	S	d	11
Kiran	A	C	ra	90	320	30	40	100	180	320	b	S	f	S	j		S	S	d	8
William	C	D	ra	90	360	45	80	160	90	10	b	c	S	S	s	L	s	p	b	9
Michael		B	90		60	80	100				S					S			b	2
Paul.	C	D	ra	90	360	30	60	110	80	15	big	S	S	S	S	big	s	S	e	8
Alex	C	D		90	360	60	80	110	94	8	S	c	S	g	j	S	s	S	b	8
Devraj																				0
23																				
ACCEPTABLE RESPONSES	14	11	13	12	13	6	5	6	2	3	8	8	12	9	5	8	3	12	6	156
% FACILITY	61	48	57	52	57	26	22	26	9	13	35	35	52	39	22	35	13	52	26	

Key.

- S represents the response 'SAME'
- big represents the response 'BIGGER'
- ra represents the response 'RIGHT ANGLE'
- rat. represents the response 'RIGHT ANGLED TRIANGLE'
- rt. represents the response 'RIGHT TRIANGLE'
- represents an omission

Notes

The ranges of acceptable responses for item 5 are:

5a	40-50
5b	70-85
5c	150-170
5d	260-280
5e	330-355

TABLE 7

EACH PUPIL's RESPONSES TO THE ANGLES TEST ITEMS: CLASS 1P

Correct responses are in RED.

ITEM NUMBER

	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7	
CORRECT	C	C	ra	90	360	45	72	152	267	343	S	S	S	S	i	S	n	S	d	19
PUPILS																				PUPIL'S SCORE (mean 6·5)
Juanita	C	C	ra	90	180	60	10	150	100	10	S	S	S	S	S	S	S	S	a	10
Serena	C	C	acute	90	360		80				b	c	S	g	i	k	s	S	c	8
Susana	C	C	acute	90	360	20	80	180			big	S	S	S	big	S	s	s	e	10
Rifait	C	D	tr	1	3	4	4	6	3	4	S	c	S	g	i	k	n	p		5
Jackie	C	D		90	12	20	14	6	9										a	2
Manama	C	D	90°	90	180	25	5	110		3	b	S	f	S	i	S	n	o	e	7
Sophia	C	D		90	100	30	20	100	10	5							big		a	2
Laura	C	C	roof	2½	1	½	½	1	1	1	b	S	S	S	i	S	S	S	b	8
Leroy	C	C		180	360	60	45	25	180	379									b	3
Basil	C	D		70	180						b	c	f	g	i	k	m	o	b	2
Paresh	B	C		105							S	c	f	g	s	k	m	o	a	2
Zaid	B	C		105							S	c	f	g	s	k	m	o	d	3
Perry	C	C	ra	50	140	30	43	70	100	130	b	S	S	S	i	S	n	s	a	10
Rajan	C	D			2	50	36	70	80	20	b	c	S	g	i	k	s	S	d	6
Robert	C	C	ra	80	380	75	70	20	25	380	a	S	f	S	i	k	s	S	e	8
Colin	C	C	ra	90	390	30	75	110	280	310	b	c	S	S	i	S	n	S	e	12
Haroon	B	D	ra	90	360						S	S	S	S	i	S	S	S	a	10
Steven	C	C	ra	90	360	90		180	280	360	N	Y	N	Y	N	Y	N	Y		6
Ashley	B																			0
Raj	C	C	ra	10	180	90		180	280	360	N	Y	N	Y	N	Y	N	Y	d	5
Chris	C	C	ra	90	360	70	80	160	200	340	S	S	S	S	i	S	n	S	b	17
21																				
ACCEPTABLE RESPONSES	17	12	8	10	6	1	5	2	4	1	6	8	10	9	11	8	5	10	3	136
% FACILITY	81	57	38	48	29	5	24	10	19	5	29	38	48	43	52	38	24	48	14	

Key.

S	represents the response 'SAME'
big	represents the response 'BIGGER'
ra	represents the response 'RIGHT ANGLE'
tr	represents the response 'TRIANGLE'
N	represents the response 'NO'
Y	represents the response 'YES'
	represents an omission

Notes

The ranges of acceptable responses for item 5 are:

5a	40-50
5b	70-85
5c	150-170
5d	260-280
5e	330-355

TABLE 8

CONDENSED RESULTS FOR EACH ITEM IN THE ANGLES TEST: CLASS 1G (21 PUPILS)

Each response is followed by its frequency.

The frequencies of the correct response are in RED.

ITEM NUMBER

RESPONSE	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7
A	a	a 2	r.a. 18	90 14	360 12	45 5	80 2	160 3	270 7	340 3	a 0								a 6
B	b 1	b 2	rat.	other 5	180 2	20 3	60 1	100 1	90 2	10	b 5								b
C	c 18	c 16	r.t.	omit 2	other 6	25	65	110 4	280	15	c	3							c 1
D	d	d 1	90°		omit 1	30 5	70 1	150	other 9	300 1	d	0							d 11
OTHER	omit 2	omit 0	acute			40 1	75	155	omit 3	345	e		5						e 2
			other 3			50 1	85 9	170 2		350 5	f		0						omit 1
			omit 0			60 1	90 2	180		355 2	g			1					
						other 4	other 5	other 10		360 1	h			3					
						omit 1	omit 1	omit 1		other 10	i				11				
										omit 1	j				1				
											k					3			
											l					0			
											m						1		
											n						3		
											o							0	
											p							2	
ITEM NUMBER	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7
NUMBER OF ACCEPTABLE RESPONSES	18	16	18	14	12	7	12	5	7	10	12	14	11	14	11	12	3	15	11
% FACILITY	86	76	86	67	57	33	57	24	33	48	57	67	52	67	52	57	14	71	52
score	12	14	11	14	4	12	12	15											
big	1	1	1	0	2	2	2	1											
omit	3	3	4	3	3	4	3	3											

Key

big represents 'BIGGER', r.a. represents 'RIGHT ANGLE',
r.t. represents 'RIGHT TRIANGLE', rat. represents 'RIGHT ANGLED TRIANGLE'.

Notes

In items 3, 4 and 5 the correct response is listed first.
In item 5, other acceptable responses are also in red.

TABLE 9

CONDENSED RESULTS FOR EACH ITEM IN THE ANGLES TEST: CLASS 1BELP (87 PUPILS)

Each response is followed by its frequency.

The frequencies of the correct responses are in RED.

ITEM NUMBER

		1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7
R	F	a 2	a 2	r.a. 54	90 50	360 38	45 17	80 18	160 5	270 6	340 3	a 2								a 11
E	R	b 12	b 4	r.a.t. 3	other 23	180 13	20 7	60 2	100 5	90 9	10 9	b 33								b 22
S	E	c 64	c 48	r.t. 1	omit 14	other 21	25 2	65 2	110 8	280 4	15 3	c	27							c 4
P	Q	d 3	d 30	90° 2		omit 15	30 10	70 3	150 2	other 37	300 2	d	3							d 28
O	U	omit 6	omit 3	acute 3			40 3	75 1	155 1	omit 31	345 1	e		8						e 9
N	E			other 2			50 2	85 3	170 7		350 3	f		15						omit 13
S	N			omit 22			60 5	90 2	180 8		360 5	g			21					
E	C						other 15	other 29	other 25		other 32	h			5					
	Y						omit 26	omit 27	omit 26		omit 29	i				37				
												j				5				
												k					20			
												l					5			
												m						10		
												n						15		
												o							8	
												p							8	
												same	29	34	41	38	17	37	40	46
												big	3	5	4	2	5	2	3	3
												omit	20	18	19	21	23	23	19	22

ITEM NUMBER	1	2	3b	3c	4	5a	5b	5c	5d	5e	6i	6ii	6iii	6iv	6v	6vi	6vii	6viii	7
NUMBER OF ACCEPTABLE RESPONSES	64	48	54	50	38	22	25	15	10	7	29	34	41	38	37	37	15	46	28
% FACILITY	74	55	62	57	44	25	29	17	11	8	33	39	47	44	43	43	17	53	32

148

149

Key-

big represents 'BIGGER', r.a. represents 'RIGHT ANGLE',
r.t. represents 'RIGHT TRIANGLE', r.a.t. represents 'RIGHT ANGLED TRIANGLE'.

Notes

In items 3, 4 and 5 the correct response is listed first.
In item 5, other acceptable responses are also in red.

6.5.3 SHORTCOMINGS OF THE TEST

During the analysis of the test responses some shortcomings of the test itself became apparent.

The answer space for item 1 was too near the top of the page and was consequently overlooked by some pupils. The wording of item 3(a) allowed for responses that were too unspecific to be interpreted. The wording of item 5 was construed as requesting an approximation rather than an estimation to be made. An alternative form of wording which might be more effective is 'Guess the exact sizes of these angles in degrees'. The inclusion of 'exact' may seem unnecessary but it would make the instruction seem even less like a request for an approximation.

In item 6 the lack of a worked example or more detailed instructions caused such unspecific responses as 'bigger' and 'yes' to be given. Even the use of 'about the same size' did not prevent pupils from attempting to find angles slightly larger than each other. Improved, but brief, wording or more detailed explanation by staff administering the test is required here. Six of the eight pairs of angles in this item were equal. During the test some pupils commented that they felt that the 'same' was occurring too frequently and then sought hard to find some pairs unequal, if only slightly so. A similar number of equal and unequal pairs would increase the tendency to judge each pair individually.

It was seen that 6(i) could have been selected as larger than 6(j) even by pupils who did not recognise the reflex angle as such or understand what the arcs at the

vertices represented. Alteration of the relative positions of (i) and (j) would increase the validity of this item. The fact that the arms of 6(l) were still not sufficiently larger than the shorter arm of 6(k) and that the angles were in different orientations caused two factors to be under test at the same time. 1G pupils found equal angles in similar (6(o)(p)), identical (6(g)(h)) and opposite (6(c)(d)) orientations slightly easier to compare than those in orientations oblique to each other. As discussed in relation to the primary test results (see page 45), pupils also find length comparison of lines oblique to each other difficult. These two notions should therefore not be combined in one test item. Due to its optical illusion effect, discussed in section 6.5.1, item 6(m)(n) failed to test the objective stated. In item 6(o)(p) the small difference in orientation enabled many pupils to judge the angles as equal. As suggested in the discussion of the item a series of pairs of angles in different relative orientations to each other is required to test the objective fully and to discover exactly which differences in orientations cause problems.

6.5.4 COMPARISON OF 1G AND 1BELP RESULTS

The results of class 1G were significantly better (at the 5% level) than those of 1BELP in six items. These were items 3(b),5(b),5(d),5(e),6(a,b) and 6(c,d). The pupils of 1G were thus better at remembering the name of a right angle and at estimating angles of 78°, 267° and 343°. They were also more aware that equal angles could be marked with arcs of different

radii and that arm length did not affect angle size.

These findings must be viewed with caution as a large number of tests (19) were performed.

It cannot be inferred from the performance of 1G pupils in 6(a)(b) that they were aware of the equality of rotations represented by the arcs because the angles could have been judged to be equal by consideration of their sharpness while the arcs were ignored. Given that pupils in 1G have, however, used a rotatable protractor and that 11 of them selected the dynamic definition of angle in item 7, it is likely that many of them viewed an angle as a rotation, indicated by the arc at its vertex.

In view of the fact that 1G had been taught the angles unit only one month earlier while 1BELP had not met work on angles since primary school it is not surprising that 1G should perform better or recall items, such as the naming of a right angle. Neither is it surprising that they achieved higher results in items including reflex angles, which rarely occur drawn as static ray pairs in primary text books, but which were taught in the angles unit. The results of 1G pupils in the other three items, 5(b), 6(a)(b) and 6(c)(d) show that they have developed a deeper understanding of angle as a result of the angles unit. They are more aware of the angle attributes which affect its size, especially having rejected that of arm length which many 1BELP pupils still considered relevent. They are also more aware of the relationship between angle measure and apparent size as exhibited by their responses to item 5(b).

The result of the t test ($t = 2.902$) indicated that there was a significant difference in the mean number of correct answers obtained by pupils in 1G and in 1BELP. The teaching

of the angles unit had thus resulted in an overall improvement in the understanding of the angle concepts tested.

In the statistical analysis the effects due to guessing in multiple choice items were not considered. Pupils were informed that this was not a test of their ability and was administered only to ascertain what they had learned in primary school. They were encouraged to omit unfamiliar items, although many did not do so. It may therefore be that some of the uninterpretable, as well as the correct responses, were in fact mere guesses.

An analysis of the omissions by both pupils in 1G and 1BELP showed that a far smaller proportion of 1G pupils than of 1BELP pupils omitted each item. This reflected not only the generally higher ability of 1G pupils to answer correctly but also the greater confidence which weaker 1G pupils possessed. Omissions are usually made by weaker pupils and, as guesses were discouraged, the results show that most of the weaker pupils of 1G had sufficient confidence to make an attempt.

It was also noted that 1G pupils opted for illogical distractors far less frequently than did 1BELP pupils. This indicated that 1G pupils were making fewer guesses and were less confused about possibly relevant angle attributes in each item.

6.6 THE MEASURING ANGLES TEST

The source of items and mode of presentation of these and of the responses is explained. Each item is then described and the responses to it are analysed. Tables of results are given. The common errors are discussed and pupils' error patterns are analysed. The results of classes 1G, 1BE and 1LP are compared and the results of the statistical tests are discussed.

6.6.1 THE TEST ITEMS - OBJECTIVES, DESCRIPTION, RESULTS AND INTERPRETATION

The Measuring Angles Test consisted of three items adapted from questions asked in the primary interviews, as can be seen from the table indicating the source of each test item on page 99 .

The first two items asked whether a half protractor and a full protractor respectively had been met in primary school.

The third item consisted of six angles to be measured with half protractors by class 1LP and full protractors by classes 1G and 1BE.

For item 3 accurate measurements were noted but answers within certain limits, as outlined below, were considered acceptable. The responses of each pupil to each item are shown, grouped according to form classes, in Tables 10 to 14 on pages 155 to 159 . These tables also show the number of correct responses to item 3 given by each pupil. The results of the twenty-one pupils in 1G are condensed in Table 15 on page 160, those of the forty four pupils of 1LP in Table 16 on page 161 and those of the forty three pupils in 1BE in Table 17 on page 162. Tables 15, 16 and 17 indicate the number of correct and acceptable responses to each part of item 3, the percentage facility of each part, the number of pupils omitting each part and the frequency of the most common errors.

Each item is reproduced below. For item 3 the correct answer and range of acceptable answers is given for each part. A table of the frequency of right (R) and wrong (W) responses for classes 1G, 1LP and 1BE is given together with

χ^2 values for 1G and 1LP and for 1LP and 1BE (significant at the 5% level if $\chi^2 > 3.841$ and at the 1% level if $\chi^2 > 6.635$). The objectives tested and details of presentation are described. Noteworthy responses are described separately for each of the classes and interpretations are made.

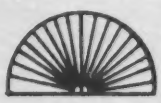
The full test in its original format is in the Appendix on page 19.

In considering the responses of 1BE and 1LP, $t = 1.661$ and thus the null hypothesis NH2 (see page 103) cannot be rejected at the 5% level.

In considering the responses of 1G and 1LP, $t = 7.358$ and thus the null hypothesis NH2 is rejected at the 5% level.

Item 1

Have you used a half-protractor like this in your primary school?



	Yes	No
1G	17	4
1BELP	73	12

Item 2

Have you used , a circle protractor, in your primary school?

	Yes	No
1G	9	12
1BELP	16	68

The objective of these items was to reveal the number of pupils who had used each type of protractor in primary school.

The items specifically used the words 'in primary school' instead of 'before' so that 1G pupils would not include their recent experiences with the full protractor.

A number of 1G pupils appear to have forgotten whether they used the full protractor in primary or secondary school as it is unlikely that as many as nine met it in primary school. It can be seen that the majority of pupils say that they have met a half protractor in primary school. In analysing responses to item 3, the responses of the pupils of 1LP who say that they have not met a half protractor are included because there is a small number of such pupils and because their responses to item 1 may have been incorrect due to the poor illustration of the half protractor. The responses to items 1 and 2 have thus only been used to obtain the approximate overall proportions of pupils who have met each type of protractor and not in order to relate the response of individual pupils to their answers to item 3.

Item 3

Use the protractor you have been given to measure these angles accurately. Write their sizes inside the loops.

This item consisted of six parts in which the sizes of a variety of angles in different orientations were to be measured. It was not the intention to test accuracy but to test the method of measurement. Consequently a range of answers was acceptable for each part. The statistics given for each part of the item relate to the number of acceptable responses.

The pupils were asked to write their answers inside the loops to help to draw their attention to the part of the angle to be measured. Many pupils did not do this, preferring to write their answers somewhere alongside the angle.

The arm lengths of most angles were unequal, the shorter one not reaching the scale marking on either protractor, to encourage the positioning of this along the zero line and consequently a variety of clockwise and anticlockwise measurements to be made. With the full protractor this was intended to provoke measurements using both the black (clockwise) and red (anti-clockwise) scales. The fact that the zero line is marked on the protractor and that the arrow is long enough to reach the scale meant that not all measurements were made in the sense intended.

In the discussion the statement 'between a and b' is taken to include the values a and b.

(a)

	R	W	
1G	21	0	$\chi^2 = 19.389$
1LP	19	25	
1BE	23	20	$\chi^2 = 0.925$

sig. at 0.1% level

Answer: 30°

Acceptable: $25^{\circ} - 35^{\circ}$

The objective was to test if pupils could measure an acute angle with its arms above its vertex and with a vertical left arm.

The arms were drawn long enough to protrude from beneath both types of protractor.

Pupils using a half protractor (1LP) with the 90° line vertical would answer 60°, those reading the wrong scale would answer 150° and those making both of these errors would answer 120°. One of the answers which could be obtained by placing the 90° line along the angle bisector is 110°.

Pupils using a full protractor and reading the wrong scale would answer 330°. Those attempting to bisect the angle with the zero line would answer in the ranges 10° to 20° and 340° to 350°.

In 1LP six pupils answered 60°, one 150°, one 120° and three 110°.

In 1BE three pupils answered 330°, five answered between 10° and 20° and one answered 350°.

In 1G every pupil gave an acceptable answer.

The results of 1G pupils were significantly better than those of 1LP indicating that pupils using a full protractor with which they had been taught to measure could measure this type of acute angle better than pupils using a half protractor with which they had been taught to measure.

(b)

	R	W	
1G	18	3	$\chi^2 = 6.826$
1LP	23	21	$\chi^2 = 0.013$
1BE	23	20	

Answer: 120°

Acceptable: 115° - 125°

sig. at 1% level

The objective was to test if pupils could measure an obtuse angle with a vertical right arm.

The non-vertical arm was drawn just long enough to reach the scale markings on each protractor. The vertical arm was shorter as it was intended that this would lie along the zero line and that the angle would be measured in an anticlockwise sense.

Pupils using a half protractor (1LP) and reading from the wrong scale would answer 60°. Those attempting to place the 90° line on the vertical arm would find that the other arm was below the protractor and that the only number which could therefore be read off was 90° itself.

Pupils using a full protractor and reading the wrong scale would answer approximately 240°.

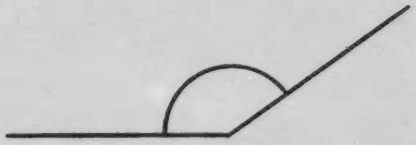
In 1LP three pupils, who had not read from the wrong scale in part (a), answered 60°. Three other pupils answered 90°, of whom one had placed the 90° line vertically in (a), and the other two had made no correct measurements in the whole item.

In 1BE five pupils answered in the range 240° to 243°. These pupils included the three pupils who had read the wrong scale in part (a).

In 1G the only three errors were answers of 240°.

The results of 1G pupils were significantly better than those of 1LP indicating that pupils using a full protractor with which they had been taught to measure could measure this type of obtuse angle better than pupils using a half protractor with which they had been taught to measure.

(c)



	R	W	
1G	15	6	$\chi^2 = 2.662$
1LP	22	22	
1BE	22	21	$\chi^2 = 0.012$

Answer: 145°

Acceptable: 140° - 150°

The objective was to test if pupils could measure an acute angle with a horizontal left arm.

Both arms were drawn just long enough to reach the scale markings on each protractor. The horizontal arm was the shorter as it was expected that the angle would be measured in a clockwise sense starting from this arm. This orientation is the most common in primary school texts, in which mainly half protractors are used. The range of acceptable answers was proportionately narrow to exclude answers such as 155° which would arise from interpolation made in the wrong sense.

Pupils using a half protractor (1LP) and reading from the wrong scale would answer 35°. Those interpolating in the wrong sense would answer approximately 155°.

Pupils using a full protractor and reading from the wrong scale would answer approximately 215°. Those interpolating in the wrong sense would answer approximately 155° and those making both errors would answer 225°.

In 1LP three pupils answered between 30° and 36°, of whom one had also read the wrong scale in part (a). Two pupils answered 155° and one 154°.

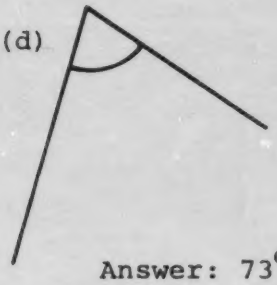
In 1BE six pupils answered between 210° and 220° of whom five had also read the wrong scale in part (b). No

pupils indicated interpolation in the incorrect sense by answering 155° but one pupil did answer 150½°, indicating both this and deeper problems in reading scales.

In 1G one pupil answered 156°, four between 215° and 220° and one answered 225°. Only one of the three pupils who had read the wrong scale in part (b) did not repeat their error.

The pupils of 1G did not achieve significantly better results than those of 1LP but they made no unclassified errors whereas sixteen pupils in 1LP did so. This indicated that pupils in 1G were positioning their protractor correctly, if slightly inaccurately, while many in 1LP were not.

(d)



	R	W	
1G	18	3	$\chi^2 = 15.157$
1LP	15	29	
1BE	20	23	$\chi^2 = 1.395$

Answer: 73°

sig. at 0.1% level

Acceptable: 70° - 76°

The objective was to test if pupils could measure an acute angle with its arms below its vertex and with neither arm horizontal or vertical.

This angle was presented in an unfamiliar orientation. Because the arms were below the vertex neither arm would fall beneath the half protractor if it was placed in its common position, with the zero line horizontal and the 90° mark above this. A different orientation of the protractor therefore would have to be used. The arms were of visibly

unequal lengths with the shorter one almost reaching the scale markings on both protractors. This was intended to encourage a clockwise measurement. The narrow range of acceptable answers specifically excluded answers arising from interpolation in the wrong sense.

Pupils using a half protractor (1LP) and reading from the wrong scale would answer approximately 107°. Those placing the 90° line on one arm would answer either approximately 20° or 160°. An answer of 150° could be obtained from the right arm when the protractor is placed with the zero line horizontally and 90° line vertically downwards.

Pupils using a full protractor and reading from the wrong scale would answer between 285° and 290°. Those interpolating in the wrong sense would answer approximately 85°. Answers close to 172° could arise from the left arm when the protractor is placed with the zero line vertical.

In 1LP four pupils answered between 107° and 110°, two answered 18°, one answered 160° and one answered 150°.

In 1BE four pupils answered between 285° and 290°, one answered 85° and one answered each of 172° and 173°.

In 1G one pupil answered 290°, another 172° and one omitted the part.

The results of 1G pupils were significantly better than those of 1LP indicating that pupils using a full protractor with which they had been taught to measure could measure acute angles with obliquely oriented arms better than pupils using a half protractor with which they had been taught to measure.

It was noted that the exact choice of orientation of

this angle was unwise as the correct answer could be obtained from the position of the left arm when the half protractor was placed with its zero line horizontal and 90° line vertically downwards. It is unlikely that the answer, 73°, was obtained in this way as all of the 1LP pupils who gave this response performed well on the item as a whole.

(e)

	R	W	
1G	13	8	$\chi^2 = 23.247$
1LP	3	41	
1LP	3	41	$\chi^2 = 3.642$
1BE	9	34	

Answer: 232°

sig. at 0.1% level

Acceptable: $230^{\circ} - 235^{\circ}$

The objective was to test if pupils could measure a reflex angle between 180° and 270°.

The angle was drawn with unequal length arms neither of which was horizontal or vertical. The shorter arm was too short to reach the scale markings on either protractor to encourage an anti-clockwise measurement when the full protractor was being used.

Pupils using a half protractor (1LP) would need to measure the obtuse angle and subtract this value from 360° to obtain the correct answer. Those just measuring the obtuse angle correctly would answer in the range 125° to 130° and those using the wrong scale would answer in the range 50° to 55°. Answers in the ranges 20° to 30° and 150° to 160° could arise from placing the zero line vertically, in which case the 90° line would be approximately bisecting

the obtuse angle.

Pupils using a full protractor and interpolating in the wrong sense would answer 248°. Those reading from the wrong scale would answer approximately 128°. Pupils making both of these errors would answer approximately 135°.

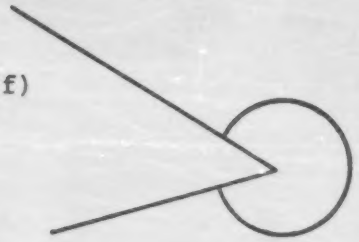
In 1LP six pupils answered between 125° and 130° while four answered between 50° and 55°. Two pupils answered 150° and two answered 20°.

In 1BE thirteen pupils answered between 125° and 130° and two pupils answered 135°. One pupil answered $130\frac{1}{2}^{\circ}$, indicating additional problems in reading scales. This pupil had also answered part (c) in a similar manner with $150\frac{1}{2}^{\circ}$.

In 1G one pupil answered 248°, one 125°, one 133° and one 136°. One pupil answered 310°, perhaps having transposed the digits of 130° when reading them upside down. Another pupil answered 23°, perhaps having omitted the final zero in the confusion of also writing the degree symbol or because the arrow of the protractor obscured the zero.

The results of 1G pupils were significantly better than those of 1LP, most of whom had not met reflex angles. It is clearly easier to measure reflex angles with a full protractor, especially for pupils who are not aware that there are 360° in a full turn. The facility for pupils in 1G is lower than that for acute and obtuse angles indicating that the angles unit should include more work on recognising and measuring reflex angles.

(f)



	R	W	
1G	17	4	$\chi^2 = 40.118$
1LP	2	42	
1BE	11	32	$\chi^2 = 7.572$

sig. at 0.1% level

Answer: 313°

Acceptable: 310° - 315°

The objective was to test if pupils could measure a reflex angle between 270° and 360°.

The angle was drawn with unequal length arms neither of which was horizontal or vertical. The shorter arm was too short to reach the scale markings on either protractor to encourage an anticlockwise measurement when the full protractor was being used.

Pupils using a half protractor (1LP) would need to measure the acute angle and to subtract this value from 360° in order to obtain the correct answer. Those just measuring the acute angle correctly would answer in the range 45° to 50° but if they were also interpolating in the wrong sense would answer 55°. Those attempting to measure the acute angle but reading the wrong scale would answer in the range 130° to 135°. Answers of approximately 43° could be obtained by placing the 90° line on one arm of the angle. Answers of 60° and 110° could arise from placing the zero line vertically, in which case the 90° line would be approximately bisecting the acute angle.

Pupils using a full protractor and reading from the wrong scale would answer approximately 47°. Those also interpolating in the wrong sense would answer 53°.

In 1LP ten pupils answered between 45° and 50°, one answered 55° and two pupils answered between 130° and 135°. Two pupils answered 42° and one 40°. One pupil answered each of 60° and 110°.

In 1BE fourteen pupils answered between 45° and 50° and one answered 55°.

In 1G one pupil answered each of 45°, 50° and 53°.

The results of 1G pupils were significantly better than those of 1LP, most of whom had not met reflex angles. Also the results of 1BE pupils were significantly better than those of 1LP indicating that pupils using a full protractor which they had not met before could measure this type of reflex angle better than pupils using a half protractor which they had been taught to use. Most of the pupils in 1LP and 1BE had not met reflex angles previously and neither class had been taught them in secondary school.

6.6.2 TABLES OF RESULTS

Tables of each pupil's responses to each test item are given, grouped according to form classes. The condensed results for each item for classes 1G, 1LP and 1BE are also tabulated.

TABLE 10

EACH PUPIL'S RESPONSES TO THE MEASURING ANGLES TEST ITEMS: CLASS 1G

Correct responses are in RED.

	ITEM NUMBER								
	1	2	3a	3b	3c	3d	3e	3f	
CORRECT			30	120	145	73	232	313	6
PUPILS									PUPIL'S SCORE (mean 4.9)
Kerry	Y	Y	30	120	145	73	248	310	5
Tina	N	N	30	120	145	75	230	313	6
Linda	Y	Y	30	120	145	73	230	312	6
Kastur	Y	N	28	119	143	73	232	315	6
Audrey	Y	N	30	120	145	70	125	45	4
Hema	Y	N	30	120	145	290	235	315	5
Shanta	Y	N	22	119	143				3
Susana	Y	N	30	120	220	70	136	50	3
Sunita	N	N	30	240	146	75	23	315	4
Mustapha	Y	Y	30	120	156	172	133	53	2
Peter	Y	Y	30	120	145	75	230	310	6
Ibrar	Y	N	30	120	215	75	230	315	5
Jason	Y	Y	30	122	146	75	230	313	6
Steven	Y	N	30	240	220	75	227	315	3
Deepak	Y	Y	29	120	144	74	234	314	6
Jay	N	Y	30	120	146	73	230	312	6
Patrick	Y	N	30	122	140	71	236	310	6
Shaun	Y	Y	30	120	143	72	233	314	6
Harish	Y	N	30	120	140	75	230	310	6
Mark	N	N	30	240	220	76	310	312	3
Paul	Y	Y	30	120	225	75	230	312	5
21									
NUMBER OF ACCEPTABLE RESPONSES			21	18	15	18	13	17	102
% FACILITY			100	86	71	86	62	81	

Key -
Y represents 'YES'
N represents 'NO'
represents an omission.

Notes

The ranges of acceptable responses for item 3 are:

3a	25-35
3b	115-125
3c	140-150
3d	70-76
3e	230-235
3f	310-315

TABLE 11

EACH PUPIL'S RESPONSES TO THE MEASURING ANGLES TEST ITEMS: CLASS 1B

Correct responses are in RED.

	ITEM NUMBER								
	1	2	3a	3b	3c	3d	3e	3f	
CORRECT			30	120	145	73	232	313	6
PUPILS									PUPIL'S SCORE (mean 2·9)
Sathana	Y	Y	30	120	145	75	125	148	4
Amarjit	N	N	40	120	141	70	130	50	3
Simone	N	N	40	120	140	70	130	50	3
Angela	Y	N	3[illegible]	120	145	70	125	47	4
Petrula	Y	N	3[illegible]	120	260	290	230	310	4
Mala	Y	N	[illegible]	120	140	70·5	130	150	4
Vanessa	Y	N	30	120	145	73	235	315	6
Rachel	Y	N	30	120	140	70	125	145	4
Jeanette	Y	N	30	120	145	75	230	310	6
Richard	Y	Y	30	120	142	72	232	310	5
Anthoney Q.	N	N	28	120	144	173	200	256	3
David	Y	N	40	90	180	45		90	0
Robert	Y	N	35	120	145	75	130	45	4
Jason	Y	Y	15	30	35	28	36	15	0
Arran	Y	N	330	240	215	29	130	55	0
Jitendra	Y	N	30	120	145	285	130	310	4
Keith	N	N	55	120	155	80	130	50	1
Antony A.	Y	N	10[illegible]	310	320	330	330	20	0
Mark	N	N	20	12	210	70	230	50	2
Roger	Y	N	30	250	220	75	230	310	4
Timothy	Y	N	350						0
NUMBER OF ACCEPTABLE RESPONSES			12	14	12	11	6	6	61
% FACILITY			57	67	57	52	29	29	

21

Key.

Y represents 'YES'
N represents 'NO'
[illegible] represents an omission

Notes

The ranges of acceptable responses for items 3a to 3f:

3a 25-35
3b 115-125
3c 140-150
3d 70-76
3e 230-235
3f 310-315

TABLE 12

EACH PUPIL'S RESPONSES TO THE MEASURING ANGLES TEST ITEMS: CLASS 1E

Correct responses are in RED.

	ITEM NUMBER								
	1	2	3a	3b	3c	3d	3e	3f	
CORRECT			30	120	145	73	232	313	6
PUPILS									PUPIL'S SCORE (mean 2·1)
Wendy	Y	N	30	120	145	75	235	315	6
Siobhan	Y	N	30	243	215	290	235	310	3
Bernadette	Y	Y	30	240	210	70	130	50	2
Razia	Y	N	30	120	140	70	125	45	4
Shabana	N	N							0
Joanna	N	N							0
Antroulla	Y	N	29	120	148	68		46	3
Julie	Y	N	30	120	145	85	135	315	4
Tracy	Y	N	30	110	140	75		80	3
Susan	N	N	20	10	8		37	5	0
Barbara	Y	N							0
Peter	Y	Y	320	350	300	300	340	360	0
Robert	Y	N	330	242	220	390	350	345	0
Edmond	Y	N							0
David A.N.	Y	N	30	120	145	73	135	310	5
Roger	Y	Y	330	240	270	70	130	45	1
David G	Y	N	70	40	100	90	100	80	0
Diego	N	N	10	20	17	18	12	12	0
Brian	Y	N	30	120	145	285	230	50	4
Ray	Y	N	40	120	145	74		47	3
Sunil	Y	N	30	118	147	73	127	47	4
saber	Y	N	30	120	145	75	225	312	5
22									
NUMBER OF ACCEPTABLE RESPONSES			11	9	10	9	3	5	47
% FACILITY			50	41	45	41	14	23	

Key:-

Y represents 'YES'

N represents 'NO'

represents an omission

Notes

The ranges of acceptable responses for item 3a are:

3a	25-35
3b	115-125
3c	140-150
3d	70-76
3e	230-235
3f	310-315

TABLE 13

EACH PUPIL'S RESPONSES TO THE MEASURING ANGLES TEST ITEMS: CLASS 1L

Correct responses are in RED.

	ITEM NUMBER								
	1	2	3a	3b	3c	3d	3e	3f	
CORRECT			30	120	145	73	232	313	6
PUPILS									PUPIL'S SCORE (mean 2·0)
Fadhia	Y	N	110	160	150	130		100	1
Boede	N	Y	30	120	121			212	2
Grace	Y		110	140	154	110	150	95	0
Shabana	Y	N	60	120	145	74		270	3
Jackie	Y	N	150	120	36	73	122	134	2
Sheila	Y	N	30	121	155	73	120	47	3
Varsha	Y	N	30	120	136	75	50	46	3
Michelle									0
Vandana	Y	N	30	60	145	74	235	126	4
Louise	Y	N	30						1
Bridgit	Y	N							0
Luke	Y	N	30	120	145	75	130	47	4
Stephen K.	N	N							0
Stephen	N	N							0
Bimal	Y	N	30	120	150	74	212	313	5
Sean	Y	N	60	90	145	150	165	47	1
Jayesh	Y	N	30	120	150	70	212	313	5
Kiran	Y	N	37	60	149	99	47	55	1
William	Y	N	30	120	145	72	128	47	4
Michael	Y	N	110	10	10	60	20	60	0
Paul	Y	N	60	120	150	110	130	42	2
Alex	Y	N	30	120	145	75	130	48	4
Devraj	Y	N							0
23									
NUMBER OF ACCEPTABLE RESPONSES			10	11	11	10	1	2	45
% FACILITY			43	48	48	43	4	9	

Key

Y represents 'YES'
N represents 'NO'
represents an omission

Notes

The ranges of acceptable responses for item 3 are:

3a	25-35
3b	115-125
3c	140-150
3d	70-76
3e	230-235
3f	310-315

TABLE 14

EACH PUPIL'S RESPONSES TO THE MEASURING ANGLES TEST ITEMS : CLASS 1P

Correct responses are in RED.

	ITEM NUMBER								
	1	2	3a	3b	3c	3d	3e	3f	
CORRECT			30	120	145	73	232	313	6
PUPILS									PUPIL'S SCORE (mean 1·9)
Juanita	Y	N	60	120	145	73	127	42	3
Serena	Y	N	34	120	140	80			3
Susana	Y	N	34	120	30	68	50		2
Rifait	Y	Y	90	90	90	80	90	80	0
Jackie	Y	Y	50	170	150				1
Mariama	Y	N	60	120	155	8			1
Sophia	Y	Y	50	170	180	81	108	65	0
Laura	Y	Y	180	170	170	160	150	110	0
Leroy	Y	N	30	120	145	73	230		5
Basil	Y	N	120	120	150			100	2
Paresh	Y	N	300						0
Zaid	Y	N	300						0
Perry	Y	N	30	60	146	70	55	50	3
Rajan	Y	Y	30	120	150		20	40	3
Robert	Y	Y	30	120	145	107	58	45	3
Colin	Y	Y	30	120	145	107	54	48	3
Haroon	Y	N	30	120	146	70	125	47	4
Steven	Y	N	60	120	35	18		130	1
Ashley	Y	Y							0
Raj	Y	N		90					0
Chris	Y	Y	30	119	145	73	231	235	5
21									
NUMBER OF ACCEPTABLE RESPONSES			9	12	11	5	2	0	39
% FACILITY			43	57	52	24	10	0	

Key

Y represents 'YES'
N represents 'NO'
represents an omission

Notes

The ranges of acceptable responses for item 3 are :

3a	25-35
3b	115-125
3c	140-150
3d	70-76
3e	230-235
3f	310-315

TABLE 15

CONDENSED RESULTS FOR EACH ITEM IN THE MEASURING ANGLES TEST: CLASS 1G

21 PUPILS USING A FULL ROTATABLE PROTRACTOR

Each response is followed by its frequency. The frequencies of the acceptable responses are in RED.

ITEM NUMBER

		1	2	3a	3b	3c	3d	3e	3f
P	F	Y 17	Y 9	30 18	120 14	145 9	73 4	232 1	313 2
E	R	N 4	N 12	28 2	119 2	140 2	70 2	125 1	45 1
S	E	omit 0	omit 0	29 1	122 2	143 3	71 1	230 8	50 1
P	O			other 0	240 3	144 1	72 1	233 1	310 4
O	U			omit 0	other 0	215 1	74 1	234 1	312 4
N	E				omit 0	220 3	75 8	235 2	314 2
S	N					225 1	76 1	other 6	315 5
E	C					other 1	290 1	omit 1	other 1
	Y					omit 0	other 1		omit 1
							omit 1		

ITEM NUMBER	3a	3b	3c	3d	3e	3f
NUMBER OF EXACTLY CORRECT RESPONSES	18	14	9	4	1	2
NUMBER OF ACCEPTABLE RESPONSES	21	18	15	18	13	17
% FACILITY	100	86	71	86	62	81

Key

Y represents 'YES'

N represents 'NO'

Notes

In item 3 the correct response is listed first.

The percentage facilities are based on the numbers of acceptable responses.

TABLE 16

CONDENSED RESULTS FOR EACH ITEM IN THE MEASURING ANGLES TEST : CLASS 1LP

44 PUPILS USING A HALF PROTRACTOR

Each response is followed by its frequency. The frequencies of the acceptable responses are in RED.

ITEM NUMBER

1	2	3a	3b	3c	3d	3e	3f
Y 39	Y 10	30 17	120 21	145 13	73 5	232 0	313 2
N 3	N 31	34 2	60 3	35 1	18 2	50 2	40 1
omit 2	omit 3	50 2	90 3	36 1	70 3	54 1	42 2
		60 6	119 1	90 1	72 1	55 1	45 1
		110 3	121 1	140 1	74 3	58 1	46 1
		120 1	170 3	149 1	75 3	125 1	47 5
		150 1	other 3	150 7	80 2	127 1	48 2
		300 2	omit 9	154 1	107 2	128 1	50 1
		other 3		155 2	110 2	130 3	other 14
		omit 7		180 1	other 7	150 2	omit 15
				other 5	omit 14	230 1	
				omit 10		231 1	
						235 1	
						other 10	
						omit 18	

ITEM NUMBER	3a	3b	3c	3d	3e	3f
NUMBER OF EXACTLY CORRECT RESPONSES	17	21	13	5	0	2
NUMBER OF ACCEPTABLE RESPONSES	19	23	22	15	3	2
% FACILITY	43	52	50	34	7	5

Key.

Y represents 'YES'

N represents 'NO'

Notes

In item 3 the correct response is listed first.

The percentage facilities are based on the numbers of acceptable responses.

TABLE 17

CONDENSED RESULTS FOR EACH ITEM IN THE MEASURING ANGLES TEST : CLASS 1BE

43 PUPILS USING A FULL ROTATABLE PROTRACTOR

Each response is followed by its frequency. The frequencies of the acceptable responses are in RED.

ITEM NUMBER

	1	2	3a	3b	3c	3d	3e	3f
RF	Y 34	Y 6	30 20	120 22	145 12	73 3	232 1	313 0
ER	N 9	N 37	28 1	90 1	35 1	18 1	125 4	45 3
SAE	omit 0	omit 0	29 1	118 1	140 5	70 8	127 1	46 1
PNQ			35 1	240 3	141 1	70·5 1	130 8	47 3
ODU			40 4	other 11	142 1	74 1	135 2	50 7
NE			330 3	omit 5	144 1	75 7	230 5	310 7
SN			other 9		147 1	80 1	235 3	312 1
EC			omit 4		148 1	285 2	other 10	315 3
Y					180 1	290 2	omit 9	other 13
					210 2	other 11		omit 5
					215 2	omit 6		
					220 2			
					other 8			
					omit 5			

ITEM NUMBER	3a	3b	3c	3d	3e	3f
NUMBER OF EXACTLY CORRECT RESPONSES	20	22	12	3	1	0
NUMBER OF ACCEPTABLE RESPONSES	23	23	22	20	9	11
% FACILITY	53	53	51	47	21	26

Key

Y represents 'YES'

N represents 'NO'

Notes

In item 3 the correct response is listed first.

The percentage facilities are based on the number of acceptable responses.

6.6.3 ANALYSIS OF ERRORS

The errors made using each type of protractor are considered separately as many of them are naturally different. A common error is the use of the wrong scale which indicates that the apparent size of the angle has not been considered. When this error occurs in the measurement of a reflex angle using a full protractor it is not clear whether the pupil is attempting to measure the non-reflex part of the angle because they cannot conceive of a reflex angle or just reading the wrong scale. Another error made with both protractors is that of interpolating in the wrong sense which arises because due attention has not been paid to the sense of the scale being used.

Tables 18 and 19 which show the frequency of errors made with the half protractor and full protractor are on pages 164 and 165 respectively. The pupils making each type of error are discussed below.

TABLE 18

Table Showing The Frequency of
Errors Made In The Measuring Angles Test

Using A Half Protractor - Class 1LP (44 pupils)

	Response	Item Part					
		a	b	c	d	e	f
Error 1	wrong scale	1	3	3	4	4	2
Error 2	90^0 vertical	6	3		1		
Error 3	errors 1 and 2	1					
Error 4	90^0 on one arm				3		3
Error 5	interpolation sense			3			1
Error 6	90^0 bisecting angle	3					
Error 7	90^0 horizontal					4	2
Error 8	non-reflex part measured correctly					6	10
	other errors	7	6	6	7	9	9
	omissions	7	9	10	14	18	15
	correct	19	23	22	15	3	2

Notes:

1. In parts (a) and (b) errors 2 and 4 are indistinguishable, they are listed under error 2.

2. In parts (e) and (f) errors 6 and 7 are indistinguishable. They are listed under error 7.

3. In classifying the errors made with the reflex angles in parts (e) and (f), the same type of error made in attempting to measure either the reflex or non-reflex part of the angle were grouped together.

TABLE 19

Table Showing The Frequency of
Errors Made In The Measuring Angles Test

Using A Full Protractor - Classes 1G (21 pupils)
and 1BE(43 pupils)

	Response	Item Part											
		a		b		c		d		e		f	
Error 9	wrong scale	3	*3*	5	*4*	6	*1*	4	*1*	13	*2*	14	
Error 10	interpolation sense				*1*	1		1	*1*				
Error 11	errors 9 and 10				*1*				*2*	3	*1*	1	
Error 12	0^0 vertical						*1*	2					
Error 13	0^0 bisecting angle	6											
	other errors	7		10		9		10	*3*	9		12	
	omissions	4		5		5	*1*	6	*1*	9		5	
	correct	*21*	23	*18*	23	*15*	22	*18*	20	*13*	9	*17*	11

Key

- *(italics)* 1G responses
- 1BE responses

Errors with the half protractor - 1LP

Error 1 - Wrong Scale

Altogether eleven pupils made this type of error, one three times, four twice and six once. It would be unusual for this error to be made in every part of the item as the angles were differently oriented, requiring measurement in different senses. No pupil made this error in order to answer always with an acute or with an obtuse angle. All pupils making this error also made other types of error, although in three cases it was only the correct measuring of the non-reflex angle in part (f).

Errors 2 and 4 - 90° line vertical and 90° line on one arm

Ten pupils made these types of errors which are indistinguishable in the responses to parts (a) and (b). One pupil made this error in parts (a), (b) and (d) but answered part (c) correctly, indicating that the 90° line had been consistently placed vertically. Three pupils who made an isolated error of this type scored zero on the whole item appearing to have made inconsistent errors.

Two pupils made this error in parts (a) and (d) indicating that the 90° line had been placed on one arm. In addition they had both measured only the obtuse angle (b) correctly and had omitted (e). One had also interpolated incorrectly in (c) and omitted (f) while the other had used the wrong scale in both (c) and (f).

Two pupils made this error in parts (a) and (f) indicating that the 90° line has been placed on one arm. They both answered (b) and (c) correctly and measured the non-reflex angle in (e). One measured (d) correctly while the other used the wrong scale.

Error 5 - Interpolation sense

This type of error was only apparent in the responses of three pupils to part (c) and of one pupil to part (f). Of these pupils, one answered all other parts correctly; one made a 'wrong scale' error and two other unclassified errors; one made two errors by placing the 90° line on an arm and omitted two parts and the fourth pupil made only one unclassified error and measured the non-reflex part of (f).

Error 6 - 90° line bisecting the angle

Three pupils answered part (a) in this way of whom two made errors in every part of the item and one gave only one correct response. Answers obtained in this way are often imprecise but it seems that these three pupils attempted to use this method in most of the other parts. Their responses are not classified due to their wide variation.

Error 7 - 90° line horizontal

This error is indistinguishable from Error 6 in parts (e) and (f). Two of the pupils who had made Error 6 in part (a) also made this type of error in parts (e) and (f) where the angle bisector is approximately horizontal. Two other pupils made this error, both of whom had also placed the 90°

line along one arm in another part.

Error 8 - non-reflex part measured correctly

Four pupils, who had made no other errors, measured both (e) and (f) in this manner. The two other pupils who measured (e) in this way measured (f) with the 90° line along one arm. Of the six others who measured (f) in this way, three had measured the non-reflex part of (e) on the wrong scale and the others had made unclassified errors in part (e), although they had made a variety of errors in other parts.

Summary

In all twenty six pupils have made one or more of the classified errors although in the case of four of these pupils it was only Error 8. Six pupils have omitted the whole item but many appear not to have followed the advice to omit unfamiliar parts. Inaccuracies of a few degrees were accepted but errors made systematically could have been obscured by inaccuracies. There was a large number of unclassified errors for each part.

The analysis shows the large number of pupils who make different types of error in different circumstances. Far fewer pupils consistently utilised one particular erroneous method even whenever it was applicable.

The most common errors were those of using the wrong scale and of placing the 90° line either vertically or on one arm of the angle.

No pupil gave six correct answers, four gave five correct answers and five gave four correct answers. No pupil

measured both reflex angles correctly and only five measured one correctly. Eight pupils made some attempt at this item but gave no correct answers.

The large number of errors made with even acute and obtuse angles highlights the depth of confusion experienced by most pupils when attempting to measure an angle with a half protractor. The obtuse angle (c) with a horizontal arm and the obtuse angle (b) which forced reorientation of the protractor prior to measurement were the parts of highest facility, approximately 50%. Clearly pupils have not related the skill of angle measurement to the concept of angle size and have a low level of proficiency in measurement, even when using the half protractor in its most familiar orientation. It was not expected that pupils would be able to measure reflex angles, which most of them had not previously met.

Errors with the full protractor - 1BE and 1G

Error 9 - Wrong scale

1BE - The occurences of this type of error with acute and obtuse angles and with reflex angles is discussed separately for the reasons discussed earlier. Ten pupils measured acute or obtuse angles on the wrong scale. All three who made this error with (a) also did so for (b) and gave either zero or one correct responses in the whole item. Four of the five who made this error with (b) also did so with (c). Two pupils made this error in all of (a), (b) and (c). Three of the four pupils making this error with (d) had made no similar error in earlier parts.

Ten pupils measured both reflex angles with the wrong scale of whom one also interpolated in the incorrect sense. Five other pupils measured (e) with the wrong scale of whom one also interpolated in the incorrect sense and another five pupils measured (f) with the wrong scale. Of the ten pupils measuring one reflex angle using the wrong scale, five measured the other correctly.

Of the ten pupils who measured acute or obtuse angles with the wrong scale, three measured both of the reflex angles correctly and three measured one reflex angle correctly.

1G - Nine pupils read from the wrong scale, three of them interpolating in the wrong sense at the same time. Only three of these nine pupils measured a reflex angle using the wrong scale.

Error 10 - Interpolation sense

1BE - Four pupils made this type of error. One pupil's only error was to use both the wrong scale and interpolation sense (Error 11) in part (e). One pupil who answered the whole item incorrectly made Error 11 in part (f). One pupil interpolated incorrectly in part (d) and made Error 11 in part (e). The fourth pupil interpolated incorrectly in part (c) and made Error 11 in part (e).

1G - Four pupils made this type of error. One pupil made both interpolation and scale errors in (e) and (f) and an interpolation error in (c). Two other pupils, with only one error in the whole item, made an interpolation error and

one other pupil made a combined scale and interpolation error in (e)

Error 12 - 0° line vertical

1BE - Two pupils made this error with (d), the acute angle with both arms pointing downwards one of whom made no other errors in this item and the other of whom made unclassified errors in (e) and (f).

1G - The pupil who had made three other interpolation errors also made this error in part (d).

Error 13 - 0° line bisecting the angle

1BE - Six pupils made this error in answering (a) of whom five gave no correct answers in the whole item. The other pupil had made an error of scale in both (c) and (f) and omitted the zero from the correct answer of 120° in (b).

1G - No pupils made this type of error.

Summary

1BE - In all nineteen pupils have made one or more of the classified errors. Four pupils omitted the whole item and ten others obtained no correct answers. Three pupils gave six correct answers, three gave five and eight gave four correct answers. The most common error was the use of the incorrect scale but it was not used consistently by any one pupil as the different orientations of the angle encouraged measurements in different senses. The interpolation error was made more consistently than any other error. There was

a large number of unclassified errors.

The large number of errors made indicated the difficulty which the pupils experienced in relating the skill of angle measurement to the concept of angle size. Given that most pupils had never before seen a full protractor they performed fairly well, and at least as well as those pupils who had measured with the half protractor which they had previously been taught to use. This suggests that the full rotatable protractor represents the essence of angle more immediately than does the half protractor. 1BE performed better than 1LP in measuring the unusually oriented acute angle (d) and, as expected, the reflex angles (e) and (f) (significantly so in the case of (f)). Clearly, though, reflex angles can be measured with a full protractor even when the notion of a reflex angle is not comprehended.

1G - In all ten pupils have made classified errors, nine of whom made at least one error of scale choice. Except for two pupils, the only other errors were due to incorrect interpolation. Ten pupils gave six correct answers, four gave five and two gave four correct answers. One pupil omitted three parts of the item.

6.6.4 COMPARISON OF 1G, 1BE and 1LP RESULTS

The results of 1G pupils were significantly better than those of 1LP in all parts of the item except (c) (at the 0.1% level in parts (a),(d),(e) and (f) and the 1% level in part (b)).

The t tests showed that there was a significant difference in the mean number of correct responses obtained by classes 1G and 1LP but not in those obtained by 1BE and 1LP. 1G pupils had been taught to use their protractors far more recently than had 1LP, and memory is clearly an important factor.

The fact that 1G made far fewer unclassified errors indicates that they could usually position their protractors correctly. Not only is the full protractor more visually symmetrical to place but also its arms can be placed on those of the angle and the rotation represented by the angle can be performed offering a connection between the angle and its measure. Once the indicator can be positioned correctly the other important factors of correct scale use and interpolation can be taught. The large number of unclassified 1LP responses did not enable the frequency with which these pupils would make these errors, once taught how to position the protractor, to be ascertained.

The provision of a full protractor clearly increased facility in measuring reflex angles but also offered greater understanding of these angles. They are hard to imagine in static ray pair drawings and circular protractors provide a way to illustrate the rotation and to compare the reflex and non-reflex angles at a vertex.

Fewer pupils in 1BE than in 1LP omitted all parts of the item and in parts (c) to (f) the number of omissions in 1BE was half or less than that in 1LP. Only one pupil in 1G omitted any part. This indicates the greater confidence offered to pupils by the full protractor, even if they have not met it before. They at least feel able to attempt a measurement, an important point when so many pupils feel anxious [21], and incapable regarding mathematics. The full protractor clearly makes reflex angles appear less mysterious but the

results show that it also assists with the understanding and measuring of other angles.

The test responses revealed the serious difficulty which pupils experienced in relating the skill of angle measurement to the concept of angle size, as described by the APU [18]. This was shown by the common error of reading the wrong scale on both protractors and by the incorrect positioning of the half protractor. This latter error indicated complete misunderstanding of angle or of which attribute was to be measured. No advancement in measuring can be made until this error is overcome.

6.7 THE INTERVIEWS

The responses of each pupil to probes regarding their answers to the secondary tests are analysed and summarised.

6.7.1 ANALYSIS OF INTERVIEW TRANSCRIPTS

M A R I A M A

Mariama correctly selected 1(c) because 'I looked at (a) and looked at the inside then I looked at (b), then (c) was the biggest circle so I knew it was biggest'.

In item 2 she had written (d) but (c) (the correct answer) was crossed out. She said 'I was going to say (c) because that's got the biggest loop but I said (d) because it has the bigger inside loop'. In primary school she has only seen loops on the inside. She clearly attempted to use the loop as her criterion in comparing angle size but was not sure how to extend this notion to reflex angles which she had not previously met.

For item 3(a) she has written 'class room wall' and when questioned qualified this with the word 'corner'. It seems

apparent that she was looking at the correct place on the wall but she has failed to specify it in writing. Responses to this item were hard to judge because of their unspecific nature.

Her answer to item 3(b) was '90 degrees angle', highlighting the qualitative nature of this name. She could not recall the name 'right angle' but remembered it when told. 3(c) was answered correctly indicating that she was probably aware that the angle was composed of ninety units called degrees.

She answered 4 incorrectly as 180°, and when shown a full protractor suggested 350° before prompting elicited the answer 'Oh! 360'. She said that she had not previously met the degree measure of a full turn.

Her answers to item 5 were all incorrect (a) = 25, (b) = 5, (c) = 110 and (e) = 3. She is asked 'Why is your answer to (b) 5? Isn't it a bit small when this (pointing to the right angle in 3) is 90°?' She replies in the affirmative and when asked for a better guess, suggests 'About 20'. She is told that it is nearer 90° and emits a shreik of amazement. She was given a full protractor, which she had never seen before, to measure (b) and immediately placed the hole over the vertex but took a little longer to align the zero line and arrow with the arms. She read off 80° correctly.

The reasons for her answers to other parts of item 5 were not probed as she had revealed great confusion regarding item 5(b). The response to 5(c) seems to be a random number larger than 90° as she was aware that the angle was obtuse. The small value for (e) suggests that she was attempting to measure

the acute rather than the reflex angle, especially as she has not met reflex angles before.

She incorrectly selects 6(b) as larger than 6(a) saying that she has looked at 'the inside' to make her decision. It is not clear whether it is the length of the red loop, area between it and the vertex or sharpness of the point which she has looked at. She is asked if the sharpness of the point matters and replies 'Yes it does matter with the point because if the point is a bit wide, that means its bigger'. She says 6(a) and 6(b) have the same sharpness. When asked again if 6(a) and 6(b) are the same size she says that they are not. Consequently in spite of realising that they are equally sharp she seems to have allowed the greater length of arc in (b) to override this as being a more important criterion for judging size. There is no evidence to show that she relates the arc to the rotation it is intended to represent, she just uses arc length as an abstract criterion.

She correctly states that 6(c) and 6(d) are the same as their points look the same and she says that the length of the arms does not matter. She incorrectly selects 6(f) as bigger stating 'if you just took (f) and put it on (e) it looks bigger. I just measured it in my mind. Then I looked at the inside loop and the point'. Here she uses the word 'loop' with 'inside' whereas 'inside' was used alone in 6(b) but probably also intended to refer to the loop as discussed above. She seemed aware that 6(f) and 6(e) could be compared by overlaying them but was unable to rotate 6(f) correctly in her mind. She became less certain as she spoke about this question, finally saying that 6(e) and 6(f) were not very different in size.

She obviously did not recognise (e) as a right angle and probably not (f) either.

She said that she decided 6(g) and 6(h) were the same by looking at the point and the loop. She had incorrectly written that (i) and (j) were the same but in the interview immediately saw that (i) was larger since having met reflex angles in item 2. She said '(j) is small inside and (i) is big outside' again referring to the loops inside and outside.

She correctly found (k) and (l) equal and (n) larger than (m) because the loop was on the outside. In the test she had incorrectly selected 6(o) as larger than 6(p) because 'the point seemed quite wide and also I looked at the loop'. She hesitated as she then saw that 6(o) and 6(p) were about equal.

When shown her answer of (e) to item 7 she laughs and says 'it's wrong' and in her attempt to make a better selection says 'It's the amount of space out of the line so not (b) or (c). It is space outside lines. It's the points. It's direction, no it isn't. That one is wrong. It's this one'. She finally points to (d) assertively. It became clear that she had initially selected (e) as it included the word 'point' and she knew that sharpness of the point was an important criterion. Her meaning of 'space outside lines' is unclear but she was not actively looking for a definition including a turn. She selected (d) partially because the other alternatives were not acceptable, but it suddenly seemed correct to her as she read it. Clearly Mariama is a logical thinker but it seems as if other pupils would also benefit from greater time spent considering this question rather than hurriedly answering it

as it is the last one.

When asked how the amount of turn was marked on a diagram she did not answer but when loops were mentioned she agreed that this was the method.

She was asked to measure Angles 6(i) (330°) with a full protractor and said 'a hundred and thirty ... I mean 35' without mentioning degrees. She was told to measure where the loop was but still turned the indicator disc through an acute angle and read off 30°.

She had used a half protractor but not a full one in primary school. In the Measuring Angles Test she had used a half protractor. Her answer for 3(a) (30°) had been 60° because she had placed the 90° line on the vertical arm. When given a full protractor she measured 3(a) as 32° and then said 'It can't be though, its 60°', and then added 'Oh this one, its the proper one' referring to the full protractor. She seemed to have said this either to please me as I seemed to be advocating the full protractor or because she felt that two sets of answers were acceptable. Maybe she perceived that the protractors measured in different units and that the full one utilised the 'correct' unit for secondary school work or perhaps she has not realised the uniqueness of a measurement in any particular unit.

In attempting to measure 3(b) (120°) she first placed the 90° line on the vertical arm but found that the protractor did not then cover the other arm of the angle and consequently no answer could be read off. After thinking for a while she realised that the protractor placed with a zero line over the vertical arm covered the other arm too. She then read off

120° correctly. Given the full protractor she quickly verified this result.

When asked how she had arrived at an answer of 155° for 3(c) (145°) she showed that she had placed the protractor correctly and 'then it came to that middle one and I counted 155 or 35'. When asked which is correct she chooses 35 not noticing that she has placed 155 midway between 140 and 150 by reading the scale in the wrong direction. When given a full protractor she measures the angle correctly as 145 which she exclaims in a surprised voice. The inconsistency has surprised her but she is still not quite sure that both protractors should give the same answer. She should have been given the half protractor to measure 3(c) again to ascertain whether or not she tried to find a way to obtain 145° with this also.

Her answer to 3(d) (73°) was 18° which she said she thought she had just guessed. It is a likely answer if the 90° line is placed over one of the arms. With the full protractor she measured it as 75°. She is asked if the angle looks less than a right angle but does not reply. It is suggested that the answer 75° looks good as it is near to 90° and the angle looks near to a right angle. She says 'Mmm' to this and adds '18 would be over here somewhere' indicating correctly a small acute angle. She seemed finally to see the relationship between common sense and measurement but only for a moment.

She had not been able to attempt the reflex angles with a half protractor. Using a full protractor she measured 3(e) (232°) as 130° because she had turned the arrow on the protractor

through the obtuse part of the angle. When this fact was pointed put to her she said 'I could have turned round that way' indicating the correct rotation. She then performed the correct rotation, read from the correct set of numbers and gave 280 as the answer, a misreading of 230°. She still did not use the word 'degrees'.

In measuring 3(f) (313°) she gave 305 and when this was said to be wrong gave 315°. She had initially read the scale in the wrong direction.

When asked to draw a free-hand angle of approximately 45° she first drew a vertical line and then drew the second arm correctly, indicating that she had remembered the size of this angle from Angles 5(a). She needed to be reminded to mark in the loop. Her efforts to draw an angle of 100° resulted in one of approximately 320° and, when this was rejected, one of approximately 340°. This may be due to a deep notion arising from work with decimals that a full turn was 100° in spite of her earlier suggestion that it was 180°, which had been made following more familiar angle items relating to right angles.

Summary

Mariama is a restless but logical child of above average intelligence and prepared to think when she has been calmed down. She understands acute and obtuse angles and can order them by their sharpness or by the loop which she also calls a circle. During the interview she was anxious to find a quick and correct way of ordering all types of angles and realised that concentrating on the size of the loop enabled her to give correct answers except in 6(a). She did not seem

to perceive the loop as representing the amount turned as is evidenced by her answer to 6(a), and her inability to measure Angles 6(i) by turning in the correct direction and in her answer to Angles item 7. It seems that she does not have a clear concept of angle but has a good memory and can store correctly methods which she has been taught in angle work but not necessarily apply them.

She was anxious to find the angles unequal in item 6 and frequently opted for one which appeared to her to be just a little larger in spite of the rewording of the item with 'about the same size' which was intended to overcome this.

She has not related her experience of degree measure in primary school to her intuitive idea of angle size except that she can recognise 90°, although not in an unusual orientation. She cannot go beyond this and recognise 45° or just more or less than 90°. She will read the number from either scale on a protractor with no thought as to which is the more appropriate and is unaware that there is a unique degree measure of an angle. She had previously used a half protractor but was more successful with the new full protractor which she also found easier. Her interpolation on the scales of both protractors was poor, as she tended to read both scales in the same direction. She positioned the half protractor with the 90° line over an arm of the angle or vertically, showing a tendency that she always wanted to keep it in the same orientation, with its straight edge from left to right, regardless of the angle to be measured. Differing results on each protractor seemed to confuse her but not to indicate an inconsistency. She does not possess the concept of angle measurement or the skill of measuring.

It seems a great pity that a fairly intelligent child has not been able to develop greater understanding of angle or angle measurement from her primary school experiences with angle.

F A D H I A

Fadhia selected 1(c) correctly. She incorrectly chose 2(d) as it was the 'widest'. When asked if she looked at the point, the loop or the end of the arms she replied 'When I measure it, I look at the little star', referring to the mid-point of the base of the half protractor. She said that she had not met reflex angles before and thus 2(d) was an expected choice.

Her written response to 3(a) was 'door' which she qualified to 'corner of the door' in the interview, again illustrating the unspecific written responses given to this item when the correct answer was known. She answered 3(b), (c) and (f) correctly saying that she had learned about these in primary school.

Her estimations for item 6 were 60°, 80°, 110°, 94° and 8°. Although the first was a little large the second was correct and showed her understanding of how to subdivide 90°. Her answer to (c) was 110°, a popular pupil response, because she could see that the angle was over 90° but not by how much. Her answers to parts (d) and (e) were good estimates of the acute rather than reflex angles highlighting the fact that she was ignoring the red loop when it was on the outside.

Her written responses to all parts of item six were either the word 'same' or 'bigger'. The omission of a worked

example had left it unclear that the name of the bigger angle was required in unequal pairs. In the interview she was asked to name the bigger angle. She incorrectly selected 6(b) as the larger and when asked how she decided upon this replied 'I looked at the corner'. When questioned whether this meant the red loop or how pointed it was she said that she looked at the red loop.

She incorrectly chose 6(c) as bigger offering no reason but correctly chose 6(e) equal to 6(f), recognising them both as right angles. She incorrectly selected 6(g) as larger than 6(h) because 'the lines are bigger'. This probably explains why she also selected 6(c) as bigger.

She incorrectly found 6(i) and 6(j) equal because 'the lines are the same length'. It was pointed out to her that the lines were the same in 6(a) and 6(b) but she had still used the loop as a criterion, and now when the loops were clearly different she was using the line length as a criterion. She appeared confused and said that she had never before met angles with loops on the outside.

She incorrectly selected 6(k) as bigger because 'one is long and one is short and in the other one both are the same'. Again this was an example, as had been shown in the primary interview, of the long and the short arm of (k) being mentioned. When told, 'but this line in 6(k) is shorter than those in 6(l)' and asked, 'so what was important to make 6(k) bigger then?'; she replied 'the other line was longer'. It seemed that the longest arm of 6(k) was the criterion although, in this test, the arms of 6(l) look clearly longer than the shorter arm of 6(k).

She first said 6(m) was larger and then 6(n) and, when asked for an explanation, replied 'It's the way I look at it. I looked at this red bit and I looked at the lines'. Perhaps she means that she chose 6(m) first because its lower arm is slightly longer than that of 6(n) and then noticed the larger loop in 6(n) and changed her mind. This may only have been possible following the explanation given about loops of reflex angles after she attempted 6(i) and 6(j).

She found 6(o) and 6(p) equal because both of them had a long line and both had a short line. She did not refer at all to the sharpness or the loop. She selected (e) for item 7 but was not able to explain why or to understand that this did not agree with her selection of 6(e) and 6(f) as equal even though they were differently oriented.

In her primary school she had used a half but not a full protractor. She had been given a half protractor for the Measuring Angles Test and had attempted all items except 3(e). She had only measured 3(c) correctly.

When asked how she had measured 3(f) (313°) she placed the half protractor so that the 90° line bisected the acute angle. She read off 110° on one side and, when asked what was on the other side, read off 60°. These readings were inaccurate but the selection of a different value for each arm did not seem to worry her. Perhaps she thought that the number on a protractor indicated the position of an arm, rather like a bearing for each arm.

In measuring 3(d) (73°) she also placed the protractor so that the 90° line approximately bisected the angle and the arm which she was looking at was directly under a multiple of

ten marking. She read off 130° as the answer. This does not tally with her correct estimation of 80° for Angles 5(b), indicating that she could recognise angles just less than a right angle and her answer to Angles 6(e) (f) which indicated that she could recognise a right angle in various orientations. Had she been asked to estimate 3(d) prior to measuring she may have been successful as her ability to estimate, as revealed in Angles 5, seemed unaffected by orientation.

She was given a full protractor and told that the top disc turned. To measure 3(a) she placed the hole correctly over the vertex, the zero line as the approximate angle bisector and the arrow over one arm of the angle. She incorrectly read off 20°. For 3(b) she used the same method and obtained 45° because the zero line was not even accurately bisecting the angle. The notion of symmetry which appeared to cause pupils to position the 90° line of the half protractor as the angle bisector have probably been carried over to measuring with the full protractor as Fadhia positioned the zero line to bisect the angle.

When asked what was the most important thing that she looked for in measuring an angle, she said that she was not sure. She had measured acute and obtuse angles in varying orientations in primary school. She stated that she found the full protractor easier to use.

Summary

Fadhia is a quiet, below average pupil. She is able to estimate the size of acute angles in degrees and knows the degree measure of a quarter, half and full turn. She is unfamiliar with reflex angles.

When comparing angles she sometimes looks at the arc size but most frequently at the arm length. She is unable to use either type of protractor correctly, often attempting to position the zero or 90^{o} line as the angle bisector and failing to do that accurately. She reads the number on either arm from either scale. She is not aware that there is a unique degree measure of an angle.

It is disappointing that a pupil able to estimate acute angles has not begun to relate the information which a protractor can provide to these estimates.

6.7.2 SUMMARY AND INTERPRETATION OF INTERVIEWS

The analysis of the interviews aided interpretation of the responses to certain test items.

Both girls had omitted 'corner' from their response to Angles 3(a) yet added it during the interview, suggesting that the omission of this word by other pupils did not mean that they were unsure of where the right angle was.

Mariama's desire to answer the parts of Angles 6 differently, six of them were pairs of equal angles, prompted her to find some angles slightly bigger, in spite of the careful wording of the question. No doubt some other pupils had assumed that there should be a greater variety of answers and had also overlooked the 'about the same size' in the question.

Mariama had initially rushed Angles 7, partly because it was the last in the test, drawing attention to the likelihood of other pupils doing likewise. Pupils may easily have opted

for imprecise definitions in the absence of the correctly worded static one they were seeking without considering the dynamic one, just as Mariama had.

Fadhia ignored red loops which were on the outside and Mariama was not sure how the required rotation was shown on the diagram. These responses showed that pupils were prepared to compare the attribute 'arc length' without ascribing any meaning to it, and experienced difficulty in extending their concepts to reflex angles. Here they invented their own rules such as, 'only consider arcs on the inside when comparing angle size' and thus rejected the reflex angles without attempting to expand their schema to include them.

The responses relating to angle measurement revealed some deep seated problems together with some advantages of the full protractor.

When first handed the full protractor Mariama immediately placed its hole over the vertex indicating how much more natural it was to use this rotating instrument than the half protractor to measure angles. She tended to retain the half protractor in its usual orientation for measuring every angle if she could whereas she was always able to position the full protractor correctly.

Both pupils exhibited problems revealed by the primary interviews in that they did not connect intuitive feelings of angle size with measurement and nor did they realise the uniqueness of measurement of a particular angle in degrees. It is not clear whether the pupils also experienced similar problems with linear measurement. Certainly great stress must be placed on these two notions with linear measurement in

primary school before they can be carried over to angular measurement.

Mariama was unable to estimate the size of any angle and Fadhia only able to estimate acute angles. Clearly more work on estimation of numbers; particularly over 100 is required in primary school.

6.8 LIST OF PROBLEMS REVEALED

In the discussion of the secondary tests and interviews various problems and misconceptions were described. Those exhibited by a number of pupils are listed.

1. Right angle not recognised in oblique orientations.
2. Name of right angle forgotten.
3. Number of degrees in a right angle not known.
4. Right angle not perceived as a special case of a more general angle.
5. The meaning of a full turn not understood.
6. Perception of angle only as static.
7. Mental comparison of angles, especially those oblique to each other, found difficult.
8. Inability to recognise reflex angles, even after teaching.
9. Obtuse angles harder to measure, estimate and compare than acute.
10. Reflex angles between 180° and 270° harder to measure, estimate and compare than reflex angles over 270°.
11. Arm length considered to affect angle size.

12. Arc length, regardless of radius, considered to affect angle size.
13. Position on page considered to affect angle size.
14. Inability to position half protractor correctly.
15. Tendency to retain half protractor with zero line horizontal.
16. Reading from wrong scale on protractor.
17. Interpolating in wrong sense on protractor scale.
18. Inability to estimate angle size conceptually and by interpolation.
19. Relationship between intuitive angle size and measurement not understood.
20. Uniqueness of the degree measure of an angle not realised.

6.9 SUMMARY OF MISCONCEPTIONS AND RECOMMENDATIONS FOR OVERCOMING THEM

A small number of pupils confused the units of measure of angle with those of temperature and the symbol for degrees with that for percentage. Wherever possible the discussion of different units and of the evolution of a form of notation (such as %) should precede their introduction in order that a frame of reference for remembering them is provided.

A few bright pupils confused the name of a right angle with that of a right angled triangle, the latter being a more recently learned concept for them. This could imply that other pupils, when they have met a right angled triangle, will fail to distinguish these two concepts. To avoid this,

emphasis could be placed on their differences at the time the right angled triangle is first named and work with triangles should not be introduced until the angle concept is firmly grasped.

The responses of 'acute' in attempting to name a right angle also emphasise the need to minimise the premature introduction of technical terms, as discussed on page 66 . When measuring with a half protractor the visual categorisation of an angle as acute or obtuse can aid the correct choice of scale, although the consideration of an angle as a rotation and counting round the scale from 0° to the required number can equally well ensure the correct choice of scale. The requirements of measuring are often given as justifications for the early use of the terms 'acute' and 'obtuse', but surely 'sharp' and 'blunt' are more easily remembered by younger pupils with small vocabularies, even if such approaches to measurement are taught. When introducing measurement using a full protractor such vocabulary is not required at all although eventually the distinction between reflex and non-reflex angles must be made. These could then be referred to as angles which 'bend back' on themselves.

I feel that it is much more important for pupils to understand basic angle concepts and to be able to recognise acute, right, obtuse and reflex angles than to be able to name them. I therefore suggest that their names can be given for pupils who wish to learn them but that discussion and exercises use non-technical words such as those described. The great importance of the right angle, however, elevates it to such a position that I feel its name should

be taught and continually empahsised, but great care must be taken to distinguish its meaning from those associated with other uses of the word 'right'.

Pupils exhibited difficulties in recognising a right angle in unusual orientations, although more 1BELP pupils responded correctly to this part of item 6 than to any other, except the final part. The large amount of primary school work on the right angle, although causing this better performance, clearly included insufficiently varied examples of right angles. The marginally better results of 1G pupils than those of 1BELP also showed that the angles unit needed improvement in this respect. The concept of a right angle in both its static and dynamic forms is central and aids the development of other angle concepts, but only if it is seen as a particular case of a more general angle rather than in isolation. Inability to recognise right angles in all orientations precludes advancement in work on both properties of shapes and angle.

Item 3(c) was an item of comparatively high facility for both classes, yet it was expected from the emphasis on the right angle and its degree measure in primary texts that more pupils in both classes would have recalled the number of degrees in a right angle. The results of 1G pupils were not very much better than those of 1BELP, indicating a shortcoming of the old angles unit. The difficulty which both classes experienced in applying their knowledge of the number of a degrees in a right angle was exhibited by the drop in facility between items 3(c) and 5(b). This indicated that it was perceived as an isolated fact, although lack of experience in

estimation could have had some effect. Some pupils were not aware that 90° was a quantity which could be subdivided while others could not relate the particular concept of a right angle to a slightly smaller angle. The parallel introduction of the general angle concept and the right angle, in both dynamic and static forms, in primary school would help to overcome these misconceptions.

The common response of 180° to item 4, as discussed in section 6.5.1 indicated that pupils were able to conceptualise rotations through multiples of right angles and to attempt to convert these rotations into degree measure. They were, however, unsure of the meaning of a full turn for the linguistic and geometric reasons described on page 38 . These findings disagree with those of the APU survey [18] (see page 24) in which many pupils were correctly able to state that there were four right angles in a full turn yet unable to translate this into degree measure.

The item of lowest facility for both classes was item 5, in which angle sizes were to be estimated. This is not surprising as work on estimation occurred rarely in primary texts and was not included in the angles unit. Although pupils in class 1G performed significantly better than those in 1BELP in three parts of the item, their highest facility was 57% which was obtained for estimating 78°. Pupils found it most difficult to estimate angles which were not near to 0° or 90°, or whose non-reflex parts were not near to 0° or 90°.

Misinterpretation of the requirements of the question and unfamiliarity with reflex angles have been suggested as factors affecting the facilities, but the deeper problems at

their root are both the inability to relate angle measure to intuitive size and the inability to estimate numbers, especially those over 100. The new angles unit must therefore include preliminary number work and perhaps estimation of lengths of lines on sparsely marked scales.

Pupils are able to order acute and obtuse angles according to size but need this ability to be systematised into ordering angles such as 30°, 60° and 90° and then estimating their sizes in degrees. They will need to be able to decide whether an acute angle is nearer to 0° or 90° and an obtuse angle nearer to 90° or 180°. Analogies with length estimation may assist in overcoming the exhibited tendency to estimate obtuse angles as slightly larger than 90°. It may be helpful to encourage recognition of specific angles other than just multiples of 90°, such as multiples of 45° and of 60°. Reflex angles must be included. Work on multiplication, fractions and angle can be brought together in both static and dynamic examples of these angles. A long and very gradual development is essential to create an understanding of the approximate size of an angle from its drawing (e.g. about half a right angle or a little more than a half turn) and of the relationship of this to its degree measure. Only when this has been established can measurement with a protractor have any meaning. The sizes of all estimated angles can be checked and the relationship between intuition and the more accurate measurement seen. A later stage in the development towards the mental estimation of angles of any size could be the provision of a full protractor to be glanced at to aid in visualising angle sizes.

Other serious misconceptions revealed related to the attributes of an angle which were considered to affect its size. Responses to item 6 showed that the most prevalent misconception was that arm length affected size. A number of pupils felt that arc length, irrespective of its radius, indicated angle size while others considered the position of angles to be important in determining their size. Some of these pupils were consistent in their misconceptions. For them, remedial work on the relationship between rotations and their static representation with the aid of drawings, tracing paper and various lengths of hinged geostrips would be able to draw attention to the relevant angle attributes and to point out why arm length, arc length regardless of radius and position were irrelevant. The type of angle for which an attribute was applicable would be discussed (e.g. that 'sharpness' could be used only for non-reflex angles). Comparison with measures, such as weight, may assist if it is pointed out that objects of greater volume may not necessarily weigh more just as angles with longer arms may not necessarily be bigger.

Other pupils were inconsistent in their misconceptions, some feeling that different attributes affected angles in different situations. The confusions that these pupils experienced about the nature of angle first had to be overcome. They also had to understand that there was a unique set of criteria for judging angle size before undertaking the remedial work outlined.

The initial work on comparison of angle size should use tracing paper, geostrips or a rotagram, an instrument similar

to an unmarked full rotatable protractor developed by Geoff Giles [11, page 156]. After this, visual comparison must be encouraged in order to overcome the difficulties which pupils had in mentally overlaying obliquely oriented angles.

The inability of many pupils to comprehend reflex angles which they had not previously met, together with the predominantly static definitions selected in item 7, indicated that many pupils possessed a narrow concept of angle. They had not related their primary school experiences of rotations to drawings of static ray pairs. This was not surprising as the primary texts failed to relate dynamic and static approaches to angle. The angles unit should therefore ensure that both approaches are understood, together with their advantages and shortcomings, and are considered to be equally important.

The lack of understanding of the relationship between intuitive angle size and angle measurement was widespread. I feel that the use of a full protractor, which more naturally conveys the essence of an angle, will assist in overcoming this. It has been seen that pupils find it easier to position correctly on the angle. Once this has been achieved the arrow can be rotated through the angle while counting along the appropriate scale to obtain its size. The dynamic form of the angle has thus been illustrated while making the measurement. The skills of scale selection and interpolation can then be developed on a firm foundation. A further advantage of the full protractor is that it enables reflex angles to be measured far more easily. It has also been seen to instill more confidence in pupils and its use by class 1G has resulted in increased proficiency in the Measuring Angles Test.

Pupils have also been unaware that there is a unique degree measure of an angle, feeling that different types of protractor can give different answers and even that the use of a different scale or of a different position with the same protractor can give different, yet equally correct, answers. This problem cannot be rectified until the concept of angular measurement has been understood. Analogy with measurement of lengths in centimetres using different, or even differently marked, centimetre rulers can be made. Explanation of the purposes of measurement, such as the recording and transmitting of information, will assist in developing an understanding of the need for universal units and of the equality of the measured sizes of an angle with all protractors.

CHAPTER 7

CONCLUSIONS

The contribution of the research to knowledge in the area is discussed. Unanticipated findings and personal benefits reaped are described. Suggestions for further research are made.

7.1 CRITICAL ASSESSMENT

The research has revealed the deep conceptual problems related to angle which most children experience. The extent of these has caused greater emphasis to be placed on their indentification and origin than on investigation of the aspects of angle which are understood.

Problems in other areas of mathematics have also been discovered. In fact difficulties observed with both measurement skills and estimation stem from problems with number.

From my experience in teaching angles certain of the findings were anticipated. These included the difficulty in recalling the name of a right angle and in recognising it in unusual orientations and the mistaken belief that arm length

affected angle size.

I had also expected that the use of full rotatable protractors would increase facility in angle measurement but was surprised, even at the primary interview stage, to see that pupils who had never met it before could measure correctly angles on which they could not even position a half protractor correctly. The increased facility in measuring and in the understanding of angle furnished by the use of the full protractor has been one of the major findings of this research.

The tests and interviews provided unexpected insight into the misinterpretation of a 'full turn'. My teaching will now include more discussion on the meaning of this term, which I had previously considered to be intuitive.

The observation that obtuse angles and reflex angles between 180° and 270° were found harder to measure and to estimate than acute and larger reflex angles has indicated a need to provide extra discussion and practice with them. I will also now avoid the inclusion of such angles in the early parts of exercises on measurement and estimation so that only one difficult concept is presented at a time.

In view of the difficulties which I had frequently seen pupils experience in trying to measure with half protractors I was surprised to find that most primary texts introduced angle measurement gradually and very well. The research was able to reveal that a major factor contributing to these difficulties was the premature introduction by primary teachers of measurement with the standard half protractor. Teachers must be strongly urged not to introduce standard protractors before

an understanding of measurement has been built up using non-standard protractors and when they do introduce them to use only full rotatable protractors.

The shortcomings of test items and experimental conditions, in which pupils from class 1G were clearly advantaged by their recent work on angles, rendered the testing of certain objectives less valid than intended. This can only be expected in small scale tests and does not invalidate the main findings of the research which were supported by in depth interviews. The advantages of being able to make definitive statements are minimal when the work undertaken is not intended to produce large scale judgments or predictions and is only to be used as a guideline at a local level. The qualitative, local findings have however been useful in forming hypotheses suitable for later large scale testing and in suggesting areas for further research.

The research as a whole has increased my perception of the inherent difficulties in the concept of angle and angle measurement. It will enable me to improve my teaching of angle and help me to interpret problems expressed and errors exhibited while teaching. A great amount of thought is still required to develop the suggestions given into items of remedial work for inclusion in the new angles unit and to interpret the value of items in the old unit in relation to the comparative test results of 1G and 1BELP. The research has, however, provided sufficient data and interpretative comment to form a sound basis for this.

7.2 SUGGESTIONS FOR FURTHER RESEARCH

Many questions were at least partially answered by the results of the research but further ones suggested themselves.

The reasons behind some of the test responses which I was unable to interpret may have been made clearer had further research time been available to interview these pupils.

There were certain objectives in the Angles test which were not fully tested by the items given. I would like to test in more detail the attributes considered to affect angle size. The greater ease with which 1G compared the two almost vertically opposite angles than the two oblique right angles led me to wonder exactly which relative positions made angles easier or harder to compare. Test items on the comparison of angle pairs in a variety of relative orientations would throw light on this.

I would also like to test class 1G at a later date, once the angles unit was not so fresh in their minds, to ascertain if the unit and especially the use of the full protractor had produced any lasting benefits.

I would like to see research on the comparison of pupils who have been taught, under identical test conditions, with the half and the full rotatable protractor respectively for both proficiency in measuring and understanding of angle concepts. I feel that the evidence so far obtained points to the superiority of the full rotatable protractor and would like to see its universal adoption if research under better conditions supports my findings.

With regard to measuring with a full protractor there may be a tendency to rotate it either clockwise or anti-clockwise or to interpolate in one particular sense. This could be investigated.

My results showed that, using a full protractor, an obtuse angle with one arm horizontal was not measured correctly by as many pupils as was an obtuse angle with one arm vertical. The orientation or apparently easier sense of measurement for each angle affects its facility. The identification of the more difficult positions could enable greater emphasis to be placed on such angles in teaching.

Responses to the secondary tests revealed greater difficulty in both estimating and measuring reflex angles less than 270° than reflex angles greater than 270°. The pupils' perception of these angles, which leads to their difficulties, could be probed in detail.

In order to evolve an effective introduction to angle in which both the dynamic and static approach were fused I would like to develop and to test pilot material.

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APPENDIX CONTENTS

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F I R S T Y E A R

A N G L E S

U N I T

NOTES ON ANGLES

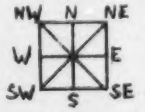
Introduce need for angular measurement for rotating objects and show that linear measurement will not be appropriate e.g. how do I describe how much I have just opened the classroom door (where could length be measured from). Rotate yourself a full turn, 1½ turns etc and describe the movement. Step 2 paces and describe that. Show the difference. Swing your arm - is it a rotation? Where's the centre of rotation? How do I describe how much I need to full wind up my watch, clock? Is it a distance? Suggest a few examples of rotating objects, car wheels, clock hands, show a rigid book page (cover) rotating as you open it. Ask the class for examples of objects which always rotate or actions which involve rotations, ask for lots of suggestions as there are so many! When appropriate ask for centres of rotation.

Now rotate 1 turn clockwise and then 1 turn anticlockwise and ask them to describe the difference - start and finish are same but what is different? Show they are inverses eg use your cylindrical pencil sharpener - screw on the lid clockwise and unscrew it anti-clockwise. Ask for other examples eg screwing and unscrewing a screw! Explain why clockwise comes from clockhands' movement, 'the way of the clock'.

Look at the homework sheet for inspiration!!

Sometimes we need to measure more accurately than whole, ½ or ¼ turns. Does anyone know how we measure amounts of turning? Degrees. Why can't we use cm? How many degrees = full turn, ½, quarter, ¼ ask round class. Show how degrees are written 360^{o}.

Making a compass

Issue their exercise books. Title 'A Compass'. They need almost a whole page. Issue 1 sheet of square gummed paper each. With the class as a whole, do the following EXACT folds very carefully. Fold in half and make crease firm with back of nail. Unfold. Fold in half , crease, unfold. Fold in half along diagonal, crease, unfold. Repeat for other diagonal. If folds meet nicely in middle and pass through corners draw over them with a ruler. Then stick paper into book leaving small gap all round. Then ask pupils to name points of compass as you indicate 

points on your drawing, then they label them around their square. Now issue ANGLE INDICATOR (also called PROTRACTOR) writing both names on the board. Hold in hand, set arrow over O. Turn ¼ turn clockwise - what colour arrow points clockwise? Black ∴ Read the black number - what is it? 90^{o}. Back to O. Turn ½ clock, ¾ clock, 1 whole clock - how many degrees - be careful to count on from 350^{o}. Repeat for anticlockwise - RED arrow implies read RED numbers. Now put on top of your compass. Arrow on O. Hole over centre exactly. Arrow pointing N. Turn clock to E - how many degrees? Turn clock to NE - make them count 45^{o}. Anticlock to NW, SW. Check get 45^{o} NOT 55^{o} and 135^{o} NOT 145^{o}. Then do angles 1 - write heading and answers only.

I expect only the brighter ones to finish sheet 7.
1st homework to be given early on and answered on sheet. Second homework at end of unit.
It is good to introduce acute, right, obtuse, reflex with their normal english meanings too and to distinguish between acute and reflex .

You can rotate a pair of geostrips to show the difference. They can draw obtuse, acute, reflex on board. Show and are equal so no need for arrow, eg measure both ways in sheet 3 or 4. Can draw angles on board, ask for name of smallest, largest, all in order WITHOUT measuring. Also can draw wedges of cheese, slices of cake for comparison. Can draw some exact angles in their book as class exercise.
All these are starting points to later lessons even if pupils are working at different rates.

ANSWERS

ANGLES 1 to 7

1. 1) 180° 2) 45° 3) 270° 4) 135° 5) 360° 6) 90° 7) 315°
   8) 225° 9) 90° 10) 270° 11) 45° 12) 225° 13) 180° 14) 315°
   15) 360° 16) 135° 16) 135° 17) 90° 18) 180° 19) 270° 20) 360°
   TOTAL 20

2. 1) 90° 2) 45° 3) 140° 4) 110° 5) 25° TOTAL 5

3. 1) 30° 2) 100° 3) 40° 4) 130° 5) 34° 6) 13° 7) 78°
   8) 32° 9) 48° 10) 65° TOTAL 10 (can be 1° out)

4. 1) 330° 2) $355\frac{1}{2}^\circ$ 3) 210° 4) 270° 5) 235° 6) 335° 7) 326°
   8) 197° 9) 320° 10) 238° TOTAL 10 (can be 1° out)

5. 1i) Yes ii) $a=b=c=d=35^\circ$ iii) $a=b=c=d=35^\circ$ iv) SAME
   2) d,a,b,c,e,f 3) $a=30^\circ$, $b=330^\circ$, $a+b=360^\circ$ 4) $a=120^\circ$, $b=240^\circ$, $a+b=360^\circ$ 5) $a=180^\circ$, $b=180^\circ$, $a+b=360^\circ$ 6) $a=220^\circ$, $b=140^\circ$, $a+b=360^\circ$
   7) $a=270^\circ$, $b=90^\circ$, $a+b=360^\circ$ 8) $a=50^\circ$, $b=310^\circ$, $a+b=360^\circ$
   9) $a=320^\circ$, $b=40^\circ$, $a+b=360^\circ$ 10) a+b is always 360°, a full turn

6. 1) acute 2) obtuse 3) acute 4) right 5) acute 6) reflex
   7) right 8) reflex 9) obtuse 10) obtuse 11) reflex 12) acute
   TOTAL 30

7. All triples of letters may be reversed.
   1) AED 2) PQR 3) ABD 4) QVR 5) LMN 6) PRQ 7) WZY
   8) EFG or DFG 9) AFC or AFD TOTAL 15

ROTATIONS HOMEWORK

1) Only NON rotations are d),h),k),n),p),q),v),w)
2) a) c b) c c) a d) c e) a f) c g) c h) c
   i) a j) a 3) ten rotations TOTAL 46

ANGLES HOMEWORK

1) g 2) e 3) b 4) c 5) 50° 6) g,a,f,b,d,c,e 7) acute
8) obtuse 9) acute 10) reflex 11) right 12) acute
13) reflex 14) obtuse 15) right 16) reflex 17) 360°
18) 270° 19) 180° 20) 90° 21) 720° 22) 3600° 23) RQP,PQR
24) DBC,CBD 25) MNJ,JNM 26) GFH,HFG 27) AFC,CFA
TOTAL 32

ROTATIONS HOMEWORK

Name _____ Form _____

(1) Which of these is a ROTATION?

If it is a ROTATION put a tick ✓ against it.
If it is NOT a rotation put a cross ✗ against it.

A car wheel turning ✓ eating a sandwich ✗

a) windscreen wipers working
b) opening a door
c) turning a door handle
d) bolting a door
e) locking a door
f) playing a record
g) pedalling a bicycle
h) pumping up a tyre
i) using a bicycle brake
j) dialling a telephone number
k) answering the telephone
l) switching on the light
m) stirring porridge
n) eating porridge
o) opening a book
p) reading a book
q) walking to school
r) looking left and then right
s) the movement of the earth
t) a bird flapping its wings
u) opening a pair of scissors
v) a bird singing
w) opening a drawer
x) a tree falling down
y) a swing swinging
z) taking out a lightbulb

(2) Say whether each of these rotations are CLOCKWISE or ANTI CLOCKWISE.

a) the hands of a clock
b) playing a record
c) turning on the cold tap
d) dialling 9
e) taking the cap off a toothpaste tube
f) putting the lid onto a coffee jar
g) drilling a hole in wood
h) traffic driving round a roundabout
i) steering a car to the left
j) bath water going down the plughole

(3) Write down TEN more objects that always rotate or actions which are rotations.

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ANGLES 1

Make sure you start with the arrow of the protractor on the 0 line.
Place the protractor so that the centre of rotation is on the centre of your compass and the 0 line and arrow are pointing NORTH.
Turn the disc of the protractor CLOCKWISE.

How many degrees does the arrow turn through in each of the following rotations?

1. North clockwise to South.
2. North clockwise to North-East.
3. North clockwise to West.
4. North clockwise to South-East.
5. North clockwise to North (one full turn).
6. North clockwise to East.
7. North clockwise to North-West.
8. North clockwise to South-West.

Start again with the arrow and the 0 line pointing NORTH.
Turn the disc of the protractor ANTICLOCKWISE.

How many degrees does the arrow turn through in each of the following rotations?

9. North anti-clockwise to West.
10. North anticlockwise to East.
11. North anticlockwise to North-West.
12. North anticlockwise to South-East.
13. North anticlockwise to South.
14. North anticlockwise to North-East.
15. North anticlockwise to North (one full turn).
16. North anticlockwise to South-West.

How many degrees are there in each of the following turns?

17. A quarter turn.
18. A half turn.
19. A three-quarter turn.
20. A full turn.

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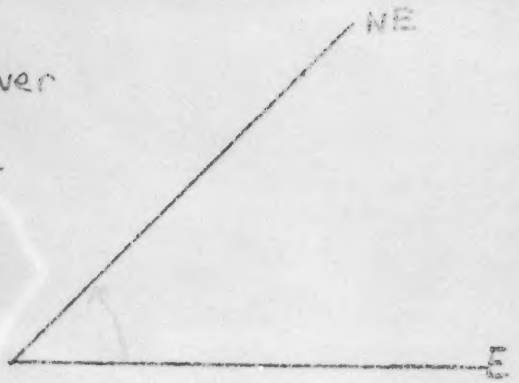
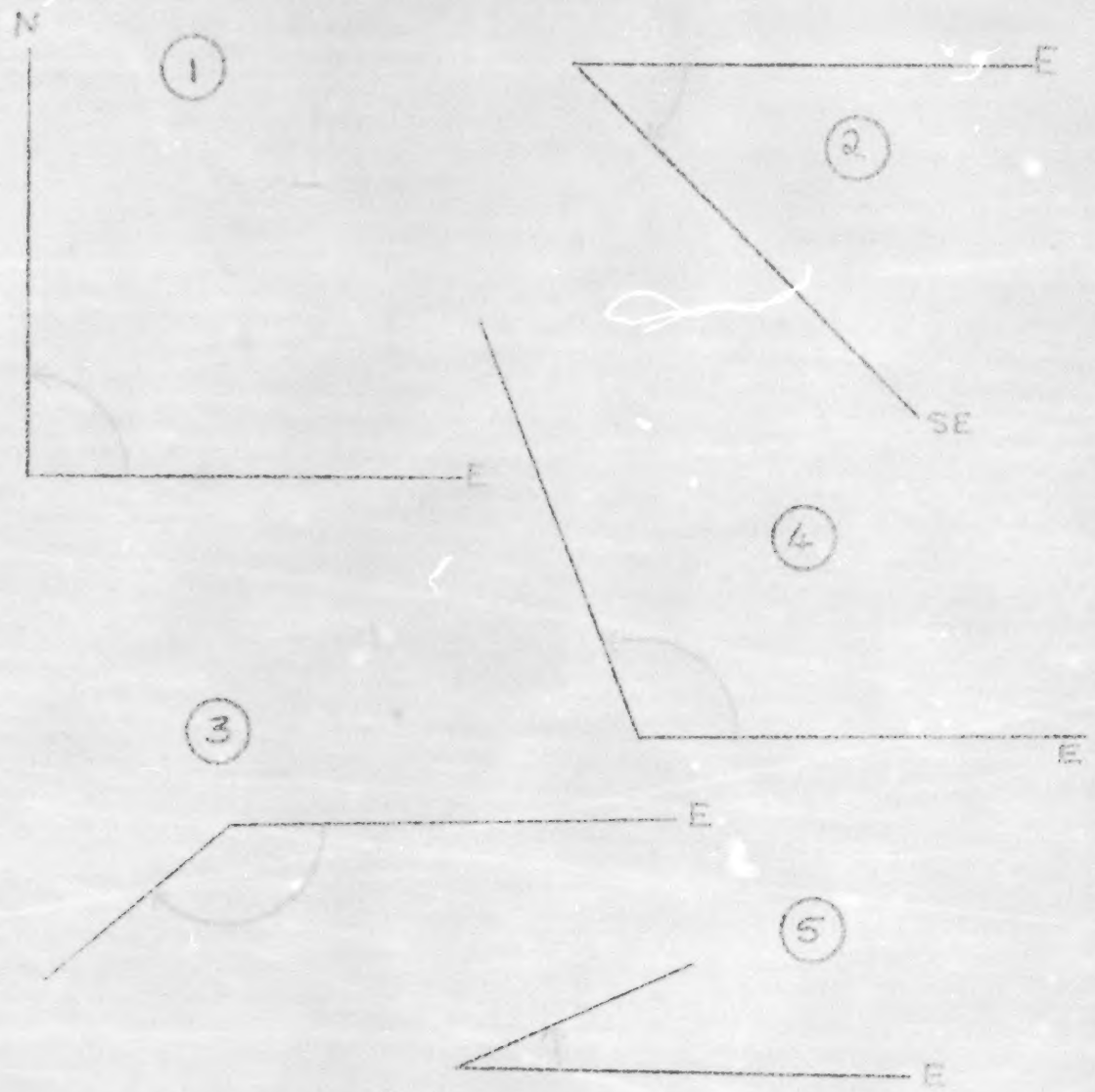
ANGLES 2



Set the arrow of your protractor over the 0 line.
Rotate the whole protractor until the arrow points EAST.
Now turn the top disc ONLY until the arrow points North-East.

How many degrees has the arrow turned through?
It should be 45°.

Measure these rotations starting with the 0 line and arrow facing EAST and turning in the direction shown.

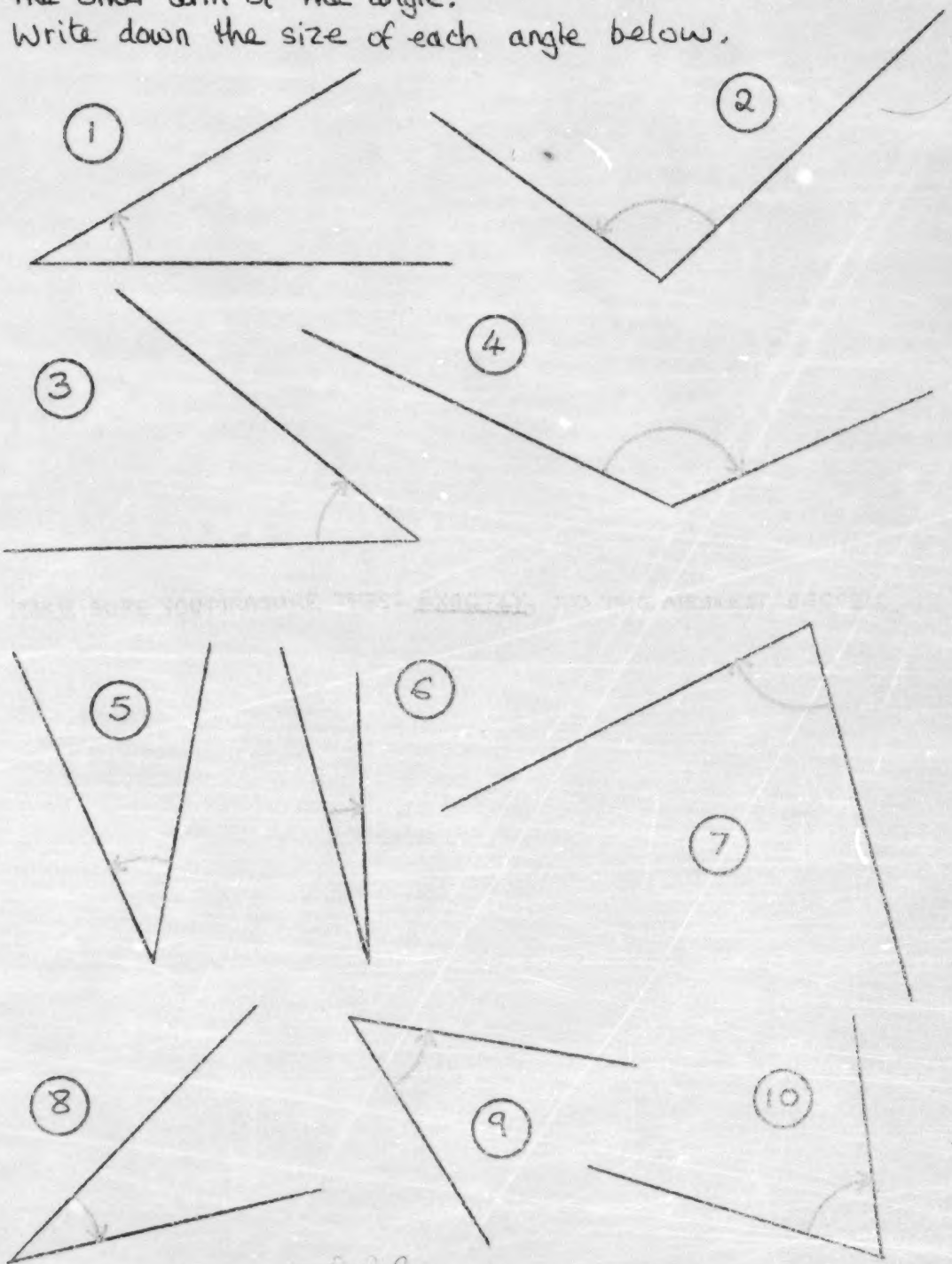


ANGLES 3

Before you measure each angle make sure the arrow of your protractor is over the 0 line. Place the centre hole of the protractor over the corner of the angle.

Start with the 0 line and arrow along one arm of the angle and turn the arrow until it lies on top of the other arm of the angle.

Write down the size of each angle below.



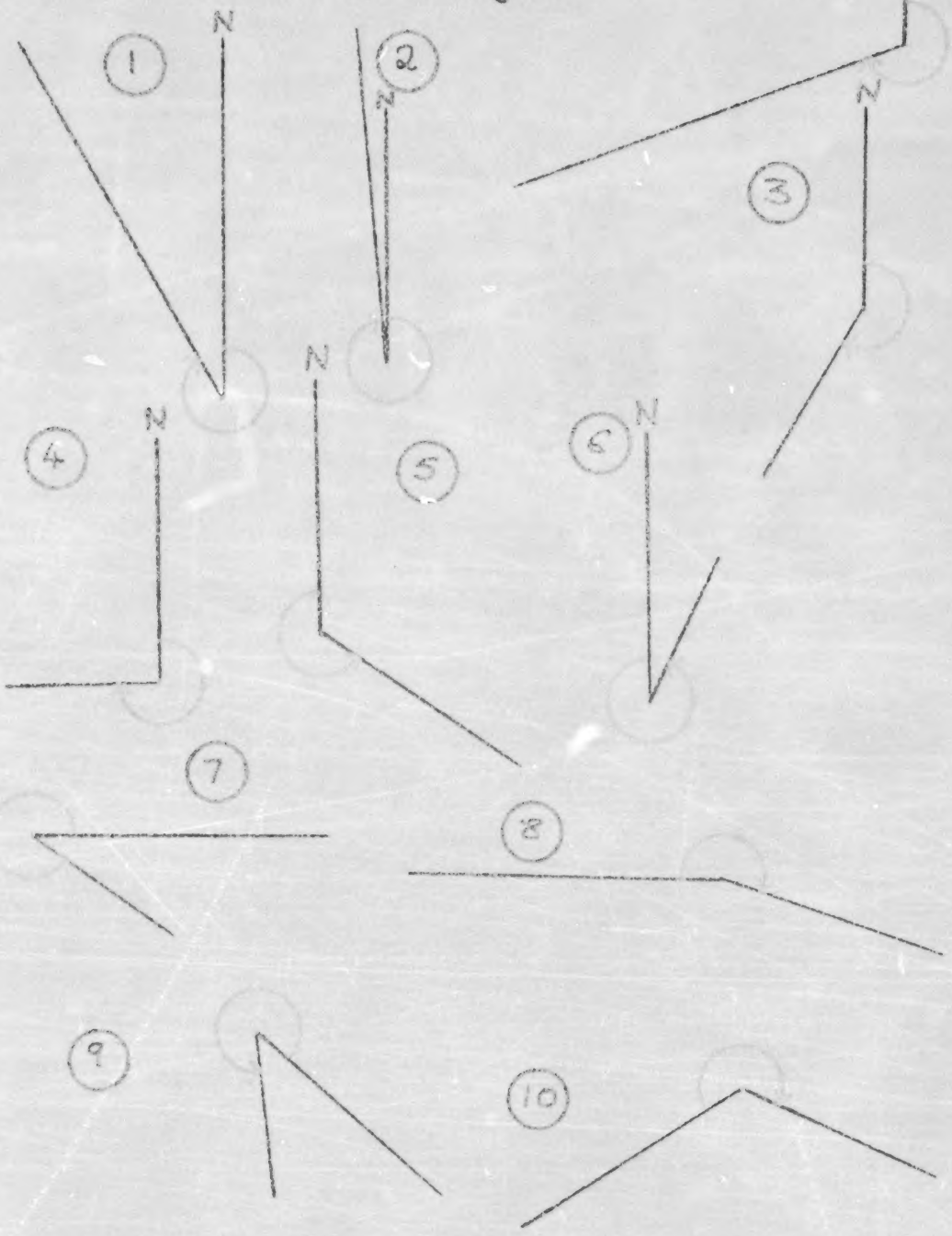
ANGLES 4 REFLEX ANGLES

Angles between 180° and 360° are called REFLEX angles.

Measure this reflex angle clockwise starting from North.

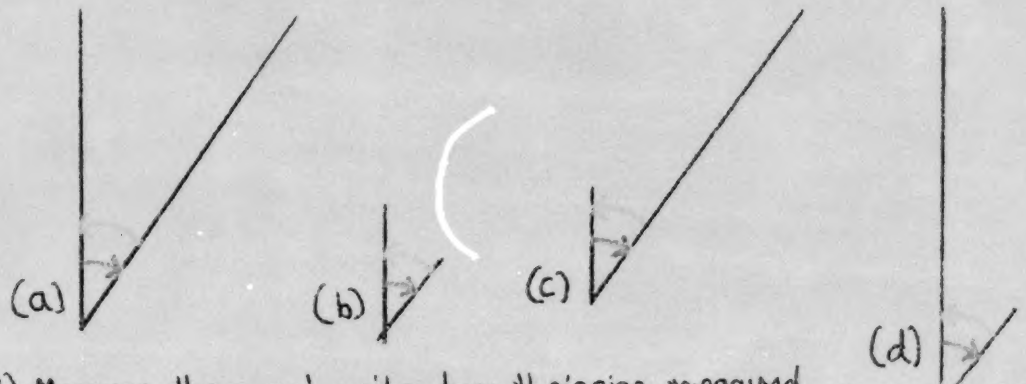
Your answer should be 250°.

Measure the other reflex angles.



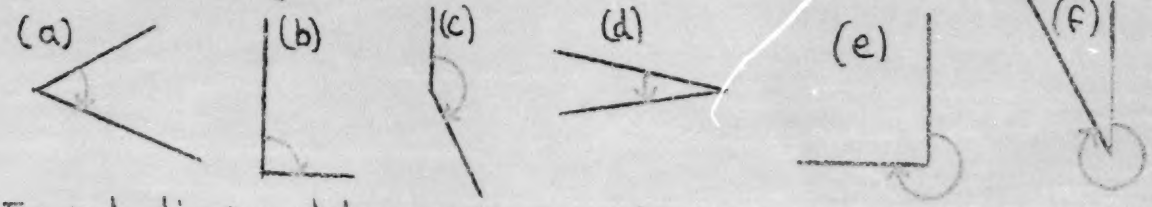
ANGLES 5

(1) i) Do you think angles (a), (b), (c) and (d) are all the same size?



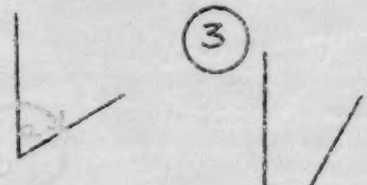
ii) Measure them and write down their size measured clockwise.
iii) Measure them again anticlockwise and write down their size.
iv) What do you notice about all of your answers?

(2) WITHOUT measuring these angles write down their letters in order of size starting with the smallest.



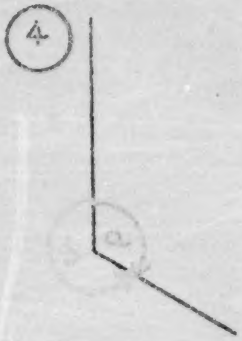
For each diagram below measure angle a and angle b. Then add your answers together as in the example.

Example
a = 60°
b = 300°
a+b = 360°



(3)

(4)



(5)

(6)

(7)

(8)

(9)

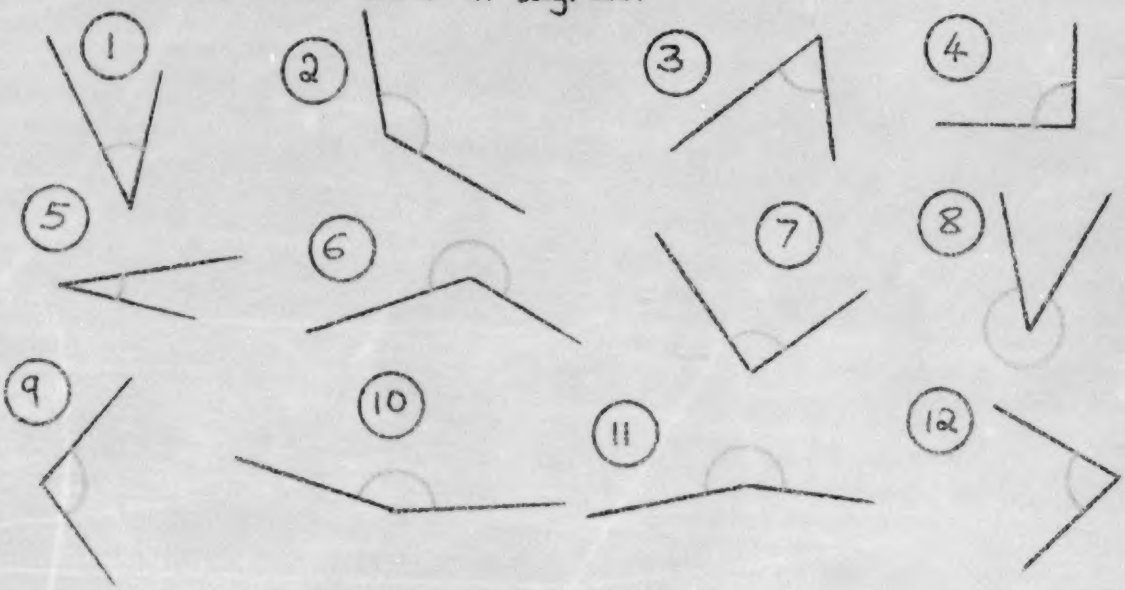
(10) What do you notice about angle (a) added to angle (b) in questions 3 to 9 ?

ANGLES 6

TYPES OF ANGLE

0° to 90°	ACUTE
90°	A RIGHT ANGLE
90° to 180°	OBTUSE
180° to 360°	REFLEX

For each of the angles drawn below write whether they are ACUTE, A RIGHT ANGLE, OBTUSE or REFLEX. Do <u>not</u> write down their size in degrees.



(13) In your book draw three different ACUTE angles. Try to draw them different ways round. Label them ACUTE ANGLES.

(14) In your book draw three right angles different ways round. Label them RIGHT ANGLES.

(15) In your book draw three OBTUSE angles different ways round. Label them OBTUSE ANGLES.

(16) In your book draw three REFLEX angles different ways round. Label them REFLEX ANGLES.

(17) Draw angles of the following sizes in your book. Label them with an arc and their size as in the example. To begin with you may find it easier to draw your first line North or East.

Example

a) 30° b) 50° c) 85° d) 125° e) 156° f) 180°
g) 203° h) 225° i) 270° j) 298° k) 317° l) 352° m) 360°

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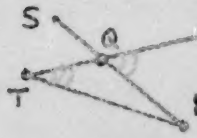
ANGLES 7 Names of angles

An angle is named by the letters along the two lines making the angle. This is angle ABC or CBA.

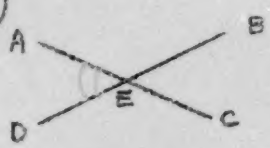


Angle PQR is marked))) . The angle marked))) can be called angle PTR or angle RTP or angle QTR or angle RTQ.

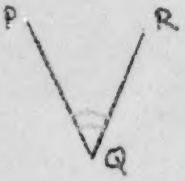
MAKE SURE THE MIDDLE LETTER is at the CORNER of the angle.

Name the angle marked))) .

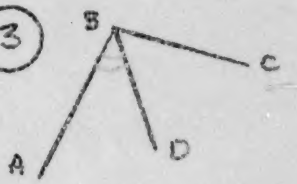
1



2



3



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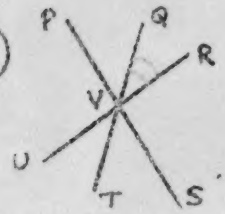

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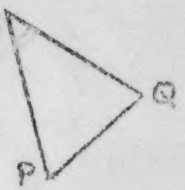

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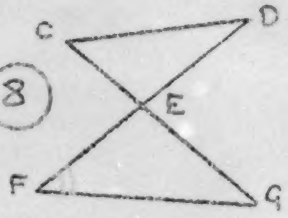

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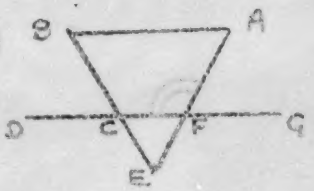
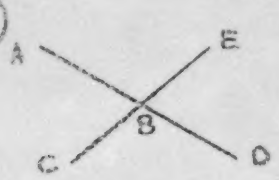


Copy these diagrams into your book (not exactly but do use a ruler). Mark angle ABC with))) .

10



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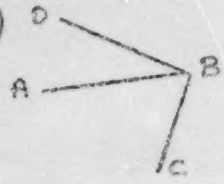
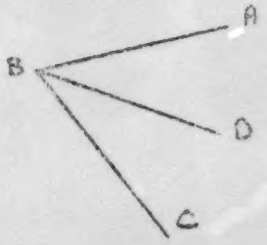


12

13



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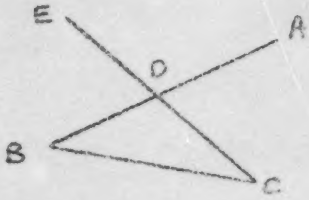

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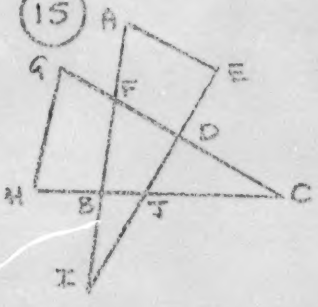


ANGLES HOMEWORK

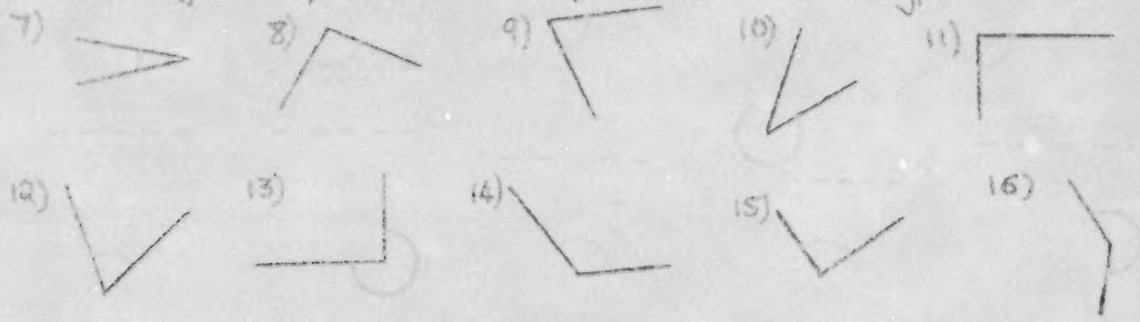
Name. Form

Look at the drawings of the angles above to help you to answer questions 1 to 6. Do NOT use an angle indicator.

1) Which is the smallest angle?
2) Which is the largest angle?
3) Which angle is nearest to 90°?
4) Which angle is nearest to 180°?
5) Which of these sizes is nearest to the size of angle f: 10°, 30°, 50°, 70°?
6) Write a list of the angles in order of size, starting with the smallest. Only write down their letters, do not guess their size in degrees.

› › › › › › .

An ACUTE angle is less than 90°. An OBTUSE angle is between 90° and 180°. A REFLEX angle is between 180° and 360°. A RIGHT ANGLE is exactly 90°. For each angle in questions 7 to 16, write down which type it is.

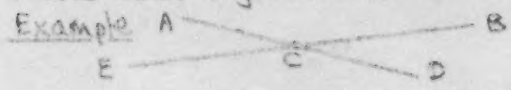


Write down the number of DEGREES in these turns in questions 17 to 22.

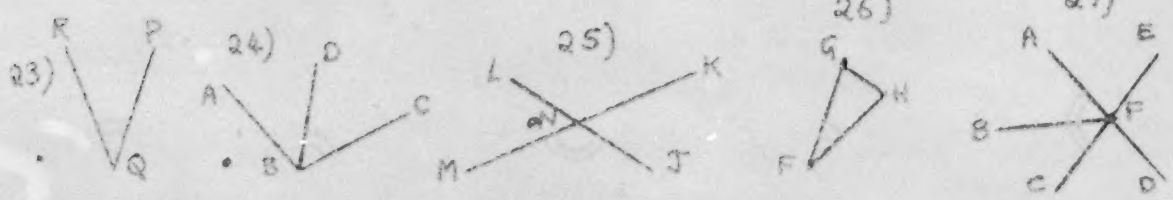
17) One full turn ____°
18) A three quarter turn ____°
19) A half turn ____°
20) A quarter turn. ____°
21) Two full turns ____°
22) Ten full turns ____°

In this diagram the angle marked can be called angle BCD or angle DCB. Name each angle marked in questions 23 to 27 in TWO different ways.

Example



B C D
D C B



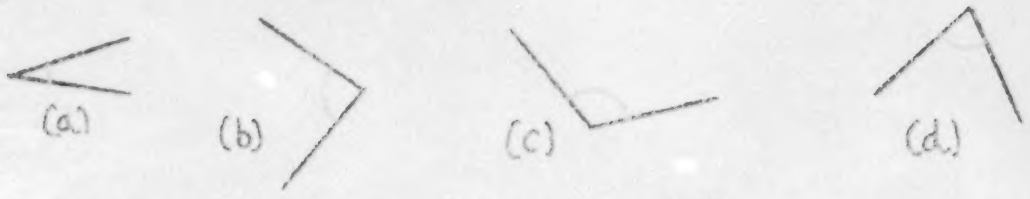
THE PRIMARY TEST

ANGLES

Name:-

(1) Which is the LARGEST of these angles?

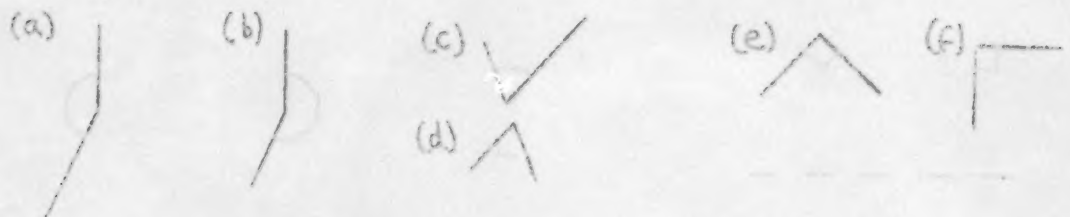
(2) Which is the SMALLEST of these angles?



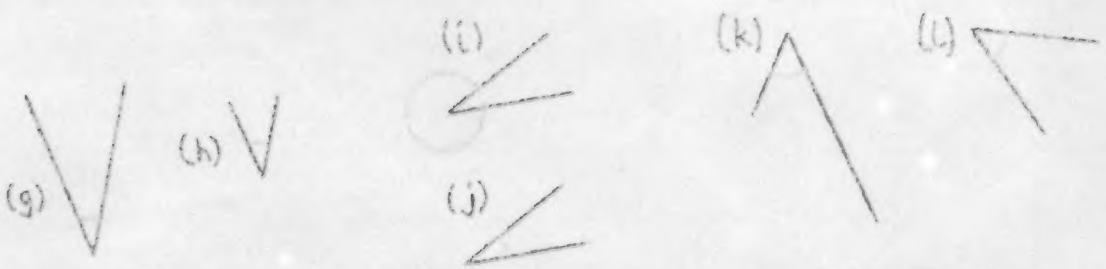
(3) Which is the LARGEST of these angles?

(4) What is the SPECIAL NAME for this angle?

(5) Which angle is BIGGER or are they the SAME size?
Answer this question for each pair of angles.



b



(6) How many degrees are there in ONE FULL TURN? _____

(7) How many degrees are there in a HALF TURN? _____

(8) How many degrees are there is this angle? _____

(9) About how many degrees are there in each of these angles?

(10) What do the 3 angles of a triangle add up to? _____

(11)

Mark with a curve 2 other angles equal to the marked one.

(12) What size is the missing angle in this triangle? _____

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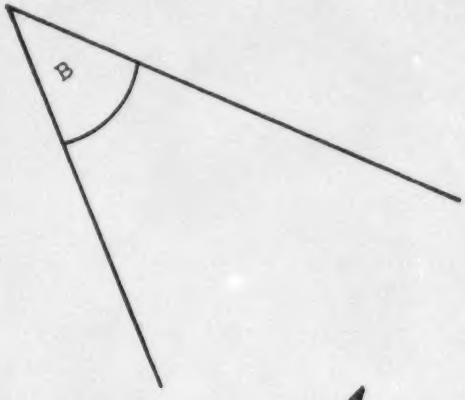
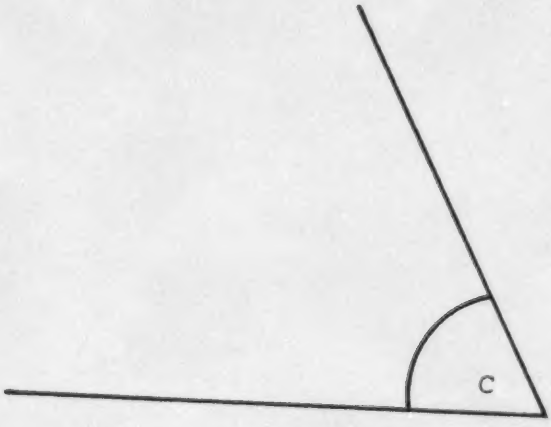
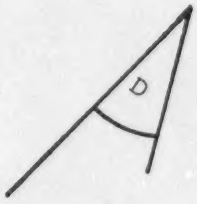
ANGLES USED IN THE PRIMARY INTERVIEWS

(ANSWERS: A=30°, B=45°, C=63°,
D=29°, E=232°, F=314°).







THE SECONDARY TESTS

MEASURING ANGLES Name:- _____ Form:- _____

Which PRIMARY SCHOOL did you go to? _____

(1) Have you used a half-protractor like this in your primary school? _____

(2) Have you used , a circle protractor, in your primary school? _____

(3) Use the protractor you have been given to measure these angles accurately. Write their sizes inside the loops.

(a)

(b)

(c)

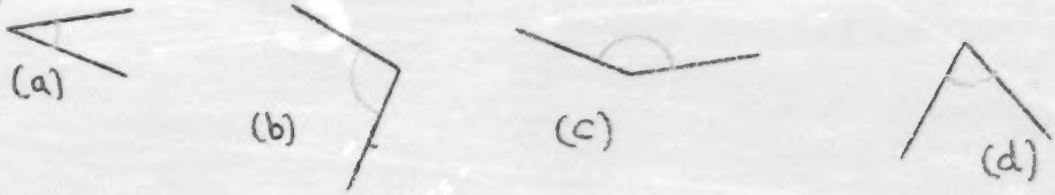
(d)

(e)

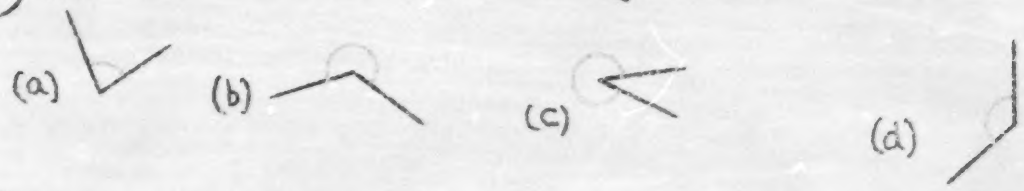
(f)

ANGLES Name:- Form:-

(1) Which is the LARGEST of these angles?



(2) Which is the LARGEST of these angles?



(3) a) Write down one place in the classroom where you can see an angle like this.

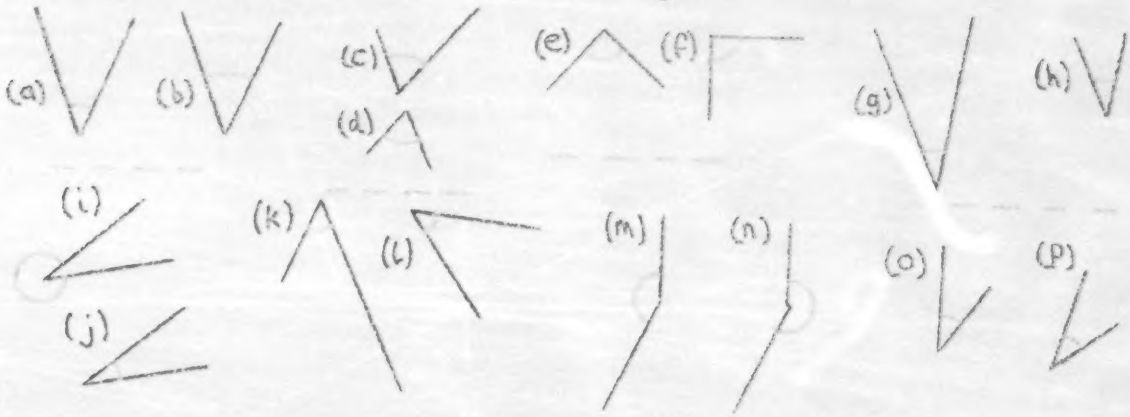
b) What is the special name for this angle?

c) How many degrees are there in this angle?

(4) How many degrees are there in ONE FULL TURN?

(5) ABOUT how many degrees are there in each of these angles?

(6) Which angle is BIGGER or are they ABOUT THE SAME SIZE? Answer this question for each pair of angles.



(7) Which ONE of these is the best way to describe an angle between two lines?

(a) The amount of space between the lines
(b) The distance between the lines
(c) The length of the lines
(d) The amount of turn from one line to the other
(e) The direction the point faces.